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Drop Size Scaling Analysis of Non-Newtonian Fluids

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DROP SIZE SCALING ANALYSIS OF NON-NEWTONIAN FLUIDS

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Summary

Current spray materials utilized by the USDA Forest Service are non-Newtonian in nature; a better understanding of their atomization behavior will improve their efficiency, efficacy, and total accountantacy and environmental fate. Thus, this study first reviews the state of Newtonian and non-Newtonian spray material characteristics, and attempts data compression with available wind tunnel data. Starting with first principles of dimensional analysis, we develop consistent nondimensional parameters that group into well-accepted numbers, such as the Reynolds and Weber numbers. We correlate the water atomization test runs from the USDA Forest Service drop size distribution data base with simple expressions that recover the details of the spray. Then we speculate upon the form of the non-Newtonian terms for the atomization of viscoelastic fluids. Finally, we suggest that a systematic laboratory benchtop examination of applicable non-Newtonian materials be undertaken, followed by a consistent set of wind tunnel tests to encompass anticipated spray materials and field conditions.

1. Introduction

The drop size distribution of spray material atomized by nozzles influences the magnitude of evaporation, spray deposition, drift, and application effectiveness. Several factors affecting the atomization process and the resulting drop size distribution include nozzle type, hydraulic line pressure, flow rate, shear across the nozzle orifice, and specific properties of the formulation. The details of the resulting drop size distribution are critical to forest and agricultural applications, where specific levels of spray material must be deposited to achieve success and avoid environmental insult.

In an effort to build a data base of typical formulations and aerial application conditions, the USDA Forest Service, and other agencies and companies, have contracted over the last several years for wind tunnel tests to determine drop size distributions of pesticides and simulant spray material atomized by nozzles. The purpose of these studies is to determine the atomization of tank mixes as influenced by hydraulic line pressure, flow rate, air velocity and shear across the atomizer, components including adjuvants (chemical, physical and biological) in the tank mix, viscosity, specific gravity, surface tension, and other atmospheric conditions. The intent is to understand the behavior of these spray materials not only to support registration and re-registration with the U. S. Environmental Protection Agency, but also to reduce the risk of material drift.

Current spray materials utilized by the USDA Forest Service are non-Newtonian in nature; a better understanding of their atomization behavior will improve their efficiency, efficacy, and total accountantacy and environmental fate. Thus, the driftable part of the atomization must be known; however, this step at present would require a wind tunnel test. Therefore, it becomes prudent to seek approaches that might reduce the need for such wind tunnel testing. One such approach is to develop a limited data base of wind tunnel tests, correlate these data, and then use them to extend predictions to conditions not tested.

A data base has already been compiled (Skyler and Barry 1990) of wind tunnel tests conducted principally at the University of California, Davis, California. Facilities to conduct wind tunnel tests must include:

1. A wind tunnel of sufficient diameter to allow horizontal expansion of the spray plume and of sufficient length and design to minimize turbulent air flow;

2. Instrumentation such as a laser to measure and count particles accurately;

3. A mechanical system for air flows up to 70 m/s;

4. Devices to position, hold, and move the atomizers;

5. An exhaust system to collect the exiting spray; and

6. A pressure metering system to transport the spray to and through the atomizer.

Other facilities known to exist include wind tunnels at Cranfield Institute of Technology, England; University of New Brunswick, Canada; USDA Agricultural Research Service, College Station, Texas; New Mexico State University, Las Cruces, New Mexico; and SpraySearch Daratech, Australia.

The purpose of the present paper is to consider the physical processes that generate the spray drop size distribution, and to deduce simple relationships between nondimensional parameters that control the breakup of a jet of liquid released through a nozzle mounted on a spray aircraft. This work extends the summary discussion on non-Newtonian fluids previously presented in Teske, Bilanin and Barry (1993).

Jet breakup atomizes the spray material into its drop size distribution. While many of the fundamental principles involved are well known, the results (for the most part) are understood only for Newtonian fluids. It is the application to non-Newtonian fluids that has received scant quantitative attention, and will be addressed in a limited sense here.

Several researchers have investigated spray distributions generated by water-based and non-Newtonian fluids (Yates, Akesson and Cowden 1984; Bouse, Carlton and Jank 1988; Bouse, Kirk and Bode 1990; and Bouse 1991 to name a few), and have also tried to correlate their data (Picot, van Vliet and Payne 1989; Womac, Williford and Hanks 1990; and Hewitt 1993). Many other studies have been conducted (such as Esterly et al. 1993), but their results are unpublished or proprietary.

Generic results from wind tunnel tests indicate the following:

1. Shear across hydraulic nozzles caused by nozzle orientation, and shear across the rotating basket cages of rotary nozzles caused by rotation, are major factors that produce small drops;

2. An increase in surface tension increases drop size;

3. Viscosity is a minor influence on atomization;

4. Rotary atomizers can become overloaded with liquid volume, resulting in larger drop size atomization even when the rotation rate is held constant;

5. Most nozzles produce a large number of small drops; however, these drops generally represent less than one percent of the total volume in the spray; and

6. Slight changes in some chemical, physical and biological properties of a tank mix can significantly alter the atomization process.

It is this last observation, and a concern for the smallest drop sizes (those that can infect the insect but also drift off-target), that underscore the problem. The need for wind tunnel evaluation of tank mixes and spray devices will continue to increase as new formulations of biological and other more environmentally acceptable pesticides are developed. Production and application costs dictate a need to improve application efficiency, efficacy, and total accountantacy and environmental fate of released material. Low volume applications require highly efficient atomization. Thus, there will be a high demand to characterize sprays in wind tunnels or through data compression. Precise atomization data will also be required for computer model simulations of aerial application.

The difficulty in discerning the atomization of current agricultural materials is that the basic spray material may rarely be treated as water, and if it is, any additives to the tank mix quickly render the spray non-Newtonian in nature. Therefore, the real problem of interest here is non-Newtonian.

2. A Non-Newtonian Primer

Rheology, the study of deformation and flow of matter, has traditionally grounded itself in Newtonian fluids. Only recently has research expanded to include non-Newtonian effects. The addition of polymers to pesticides, and the formulation of spray materials that do not use water or oil carriers, have brought home the need to investigate this extremely complicated subject further. In addition, the available literature suggests that non-Newtonian fluids cannot be approximated with Newtonian equations; small corrections to Newtonian behavior do not yield non-Newtonian behavior.

Few recent publications exist that deal specifically with non-Newtonian effects. Lee (1979) discusses some of the more intriguing properties of non-Newtonian fluids. Knell (1989) hypothesizes a representation for non-Newtonian viscosity based on Newtonian fluids, while Mansour and Chigier (1993) attempt to measure extensional viscosity, Shavit and Chigier (1993) examine dynamic surface tension, and Kitamura and Takahashi (1988) and Dooher et al. (1988) study non-Newtonian atomization effects. Chhabra (1993) examines the behavior of particles in non-Newtonian fluids; his Chapter 2 summarizes non-Newtonian fluid behavior, and contains an extensive bibliography.

An elastic fluid may be said to possess "perfect memory": as long as the material is not stressed to yield, it will return to its original state when the disturbing force is removed. Viscoelastic fluids (non-Newtonian fluids) possess what is known as "partial memory": these fluids do not necessarily return to their original state after a disturbance is applied, and then removed.

For the most part the laws of Newtonian physics are suspect for non-Newtonian fluids. Among the more dramatic effects these fluids exhibit are the following (Lodge 1964 and Bird, Armstrong and Hassager 1977 with pictures):

1. Recoil: when a non-Newtonian fluid is poured from one beaker to another, and the stream is cut by a pair of scissors, the upper portion of fluid already out of the upper beaker will retreat back into the beaker (all Newtonian fluids will fall into the lower beaker because of gravity).

2. Rod-Climbing: a non-Newtonian fluid will climb a rotating rod, contrary to centrifugal force effects (Newtonian fluids form a depression around rotating rods).

3. Die-Swell: a non-Newtonian fluid leaving a cylindrical tube through an orifice will expand to a diameter considerably larger than the tube diameter (Newtonian fluids are subject to the venturi effect, reaching a diameter smaller than the orifice diameter before expanding radially).

4. Tubeless Siphon: when a siphon is lifted out of a beaker of non-Newtonian fluid, the fluid continues to be siphoned (Newtonian fluids would stop flowing).

5. Drag Reduction: most commonly, the addition of a polymer at only 10 to 50 parts per million may reduce the drag of a fluid by up to 50 percent over its Newtonian value.

With spray materials now, as likely as not, possessing non-Newtonian behavior, it is clearly important to analyze the effects of these fluids on the generation of spray drop size distributions, and what effects they may cause beyond Newtonian behavior.

3. Constitutive Equation

Newtonian fluids (such as water) behave under deformation in the well-known linear manner:

$$\mu \frac{\partial U}{\partial r} = \tau \tag{1}$$

where μ is the absolute viscosity, $\partial U/\partial r$ is the rate of strain, and τ is the resulting fluid stress. When the fluid is non-Newtonian, Eq. (1) no longer applies. Rather, the assumption is made (Goldin et al. 1969) that the rheological properties of any incompressible isotropic viscoelastic material in the limit of very small deformations follows the constitutive equation developed by Biot (1954) and Bland (1960), written for a fluid as:

$$\mu \frac{\partial U}{\partial r} = E\,\varepsilon + \eta \frac{\partial \varepsilon}{\partial t} + \sum_{n=1}^{N} \int_{-\infty}^{t} c_n \exp\left[- \frac{(t-\tau)}{\tau_n} \right] \frac{\partial \varepsilon}{\partial \tau}\, d\tau \tag{2}$$

where U is the fluid velocity, r is a length, ε is the linear strain, E is the modulus of elasticity, η is the long-term viscous flow effect, t is time, N is the number of degrees of freedom in the fluid, c_n are amplitude coefficients, and τ_n are time constants. The quantity $\partial\varepsilon/\partial t$ denotes the rate of strain; E , η , c_n and τ_n are real and positive. c_n and τ_n are present because of the non-Newtonian nature of the material. These quantities must be a part of any analysis or interpretation of non-Newtonian fluid behavior.

4. Dimensional Analysis

Dimensional analysis is used here to infer the nondimensional groups that control the diameters $D_{0.1}$, $D_{0.5}$, and $D_{0.9}$ describing the structure of a drop size distribution (the diameters below which 10, 50, and 90 percent of the spray material is found, respectively). The principles of dimensional analysis are explained in any first book on engineering (for example, Kuethe and Schetzer 1959), and are reviewed here before proceeding.

If an algebraic equation expresses a relationship among physical quantities, the equation can have meaning only if the terms involved are dimensionally identical. The requirement of dimensional homogeneity in physical equations is useful in determining possible combinations in which variables can occur. If it is required that all terms in an equation be pure numbers (a general assumption), then the variables involved in the equation may occur only in combinations that have zero resultant dimensions. Any physical equation can in fact be expressed in terms of dimensionless combinations of the variables that influence the behavior of the equation. The formal statement of this fact is embodied in the Buckingham Π theorem; simply stated, any function of N variables:

$$\text{fcn}\,(P_1, P_2, P_3, P_4, \ldots, P_N) = 0 \tag{3}$$

may be expressed in terms of $N-K$ Π products:

$$\text{fcn}\,(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \ldots, \Pi_{N-K}) = 0 \tag{4}$$

where each Π_i parameter is a dimensionless combination of an arbitrarily selected set of K variables, and one other. K is equal to the number of fundamental dimensions required to describe the variables P_i. Here all quantities P_i may be expressed in terms of mass M, length L, and time T, and therefore $K = 3$. The arbitrarily selected set of K variables may contain any of the quantities P_i with the restriction that the K set itself not form a dimensionless combination.

This approach cannot determine the values for the constants or coefficients in the functional relationships that are developed. Experimental (in this case wind tunnel) data must be analyzed in order to deduce the values for the constants or coefficients in these relationships.

In the present problem we consider a simple jet issuing from a nozzle oriented at an angle into the free stream air flow (Figure 1). The variables that control jet breakup into drops are summarized as follows:

VARIABLE	DESCRIPTION	DIMENSIONS
D_j	jet or nozzle diameter	L
ρ	fluid density	ML^{-3}
μ	fluid viscosity	$ML^{-1}T^{-1}$
c_n	stress relaxation amplitude	$ML^{-1}T^{-2}$
τ_n	fluid relaxation time scale	T
U_j	fluid jet velocity	LT^{-1}
ρ_∞	air density	ML^{-3}
μ_∞	air viscosity	$ML^{-1}T^{-1}$
U_∞	air velocity	LT^{-1}
σ	surface tension	MT^{-2}
β	angle between airflow and jet	-
Ω	rotation rate (rotary atomizers)	T^{-1}
$D_{0.k}$	scaling diameter	L

Jet characteristics are denoted by the j subscript. The angle β is measured such that $\beta = 0$ places the jet co-flowing with the air stream. Only fluid mechanics is investigated here, including the non-Newtonian effects present with c_n and τ_n, as given in Eq. (2). Any heat transfer effects because of temperature change must be reflected in changes in the fundamental variables, and not in a temperature variable itself (temperature is not typically measured when drop size distributions are determined).

Dimensional analysis suggest that these 13 variables can form ten arbitrary parameters. One of them is the desired scaling parameter:

$$\frac{D_{0.k}}{D_j} = \text{fcn}(\ ) \tag{5}$$

where k takes on the values of 1 , 5 , or 9 . The rest of the functional forms may be obtained from previous experience and/or extended from the analysis of Lee (1979) to jet flow:

Π EXPRESSION		INTERPRETATION
$\frac{U_j}{U_\infty}$		velocity ratio
β		exit angle
$\frac{\Omega D_j}{U_j}$	Π_r	rotation rate ratio (rotary atomizers)
$\frac{\rho}{\rho_\infty}$		density ratio
$\frac{\mu}{\mu_\infty}$		viscosity ratio
$\frac{\rho U_j D_j}{\mu}$	Re	Reynolds number (aerodynamic force / viscous force)
$\frac{\rho U_j^2 D_j}{\sigma}$	We	Weber number (aerodynamic force / surface tension force)
$\frac{c_n D_j}{\mu U_j}$	Π_s	non-Newtonian stress / Newtonian stress
$\frac{\tau_n U_j}{D_j}$	De	Deborah number (liquid relaxation time / flow time)

The U_j velocities present in the above expressions are interchangeable with U_∞ through the velocity ratio. The effects of hydrostatic head (through the introduction of gravity and the Froude number) and of compressibility (through the speed of sound and the Mach number) are assumed negligible in the present analysis. For water the density ratio $\rho / \rho_\infty = 815.3$; specific gravities of spray material do not vary significantly from a value of 1.0 , implying that the density ratio need not concern us here. Likewise, the viscosity ratio $\mu / \mu_\infty = 184.2$ and is also not examined here.

The Reynolds and Weber numbers are familiar to atomization processes (see throughout Lefebvre 1989). The unnamed parameter Π_s has received no attention as far as our literature search could indicate, yet it is the parameter that reflects the effects of fluid stress. The Deborah number, on the other hand, tends to reflects the time-scale response of non-Newtonian fluids (Reiner 1964 and Metzner, White and Denn 1966). At low Deborah numbers the material exhibits a fluid-like behavior, while at high Deborah

numbers the material exhibits a solid-like behavior. An example usually cited in the literature (Lee 1979) is to move a fist slowly into containers of Newtonian and non-Newtonian liquids -- both exhibit the same effect of allowing the hand to enter. If, however, the fist is thrust quickly into the Newtonian fluid, splashing results. In the non-Newtonian fluid the surface of the material responds much like a solid surface -- the hand would not enter the material and splashing would not occur. For a Deborah number greater than about 0.05, spray droplet sizes can be ten percent larger than with Newtonian fluids.

Any expression constructed from these parameters must have a basis in Newtonian fluids. Thus, it may be seen (also from Eq. (2)) that in the Newtonian limit, $c_n = 0$ and $\tau_n = 0$.

Non-Newtonian terminology sometimes includes the expressions dynamic surface tension and elongational viscosity (briefly discussed in Lodge 1964). Purely as an exercise in recovering these types of variables, it may be seen that a product of the two non-Newtonian parameters gives:

$$\frac{c_n \tau_n}{\mu}$$

which would enable the numerator of this parameter to be interpreted as an additional viscosity parameter (with units of $ML^{-1}T^{-1}$). Also, the product:

$$c_n D_j$$

has the units of MT^{-2}, consistent with surface tension. This parameter could then be interpreted as an additional surface tension. What may happen, unfortunately, is that c_n and τ_n are related in complicated ways to dynamic surface tension and elongational viscosity. Alternately, the surface tension σ and the fluid viscosity μ may be re-interpreted in the Reynolds and Weber number regimes present here. Mansour and Chigier (1993) suggest that fluid properties should only be measured under conditions consistent with atomization, but readily acknowledge that suitable instruments are still being developed to measure the required physical properties (which in turn have not been decided upon).

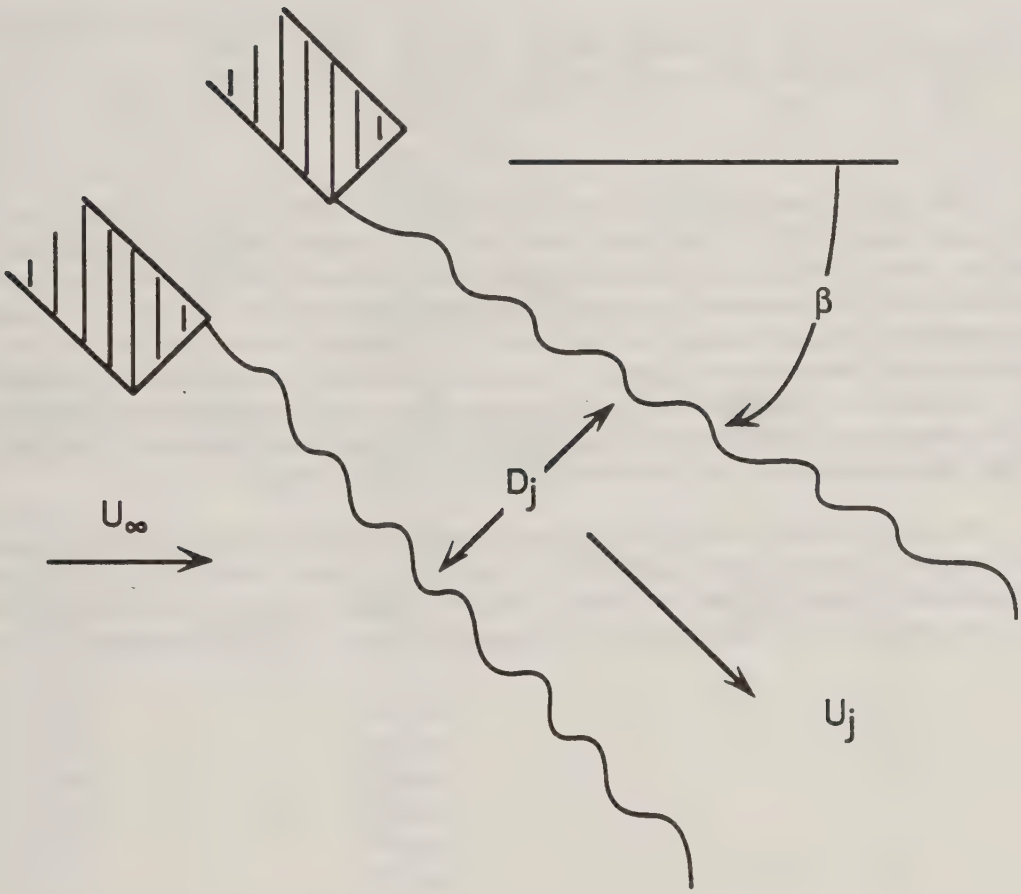

Figure 1. Schematic of jet breakup.

5. Available Data Sets

Wind tunnel data must be available to determine the constants and coefficients forming the anticipated drop size equation:

$$\frac{D_{0.k}}{D_j} = \text{fcn}\left(\frac{U_j}{U_\infty}, \beta, \frac{\Omega D_j}{U_j}, \frac{\rho U_j D_j}{\mu}, \frac{\rho U_j^2 D_j}{\sigma}, \frac{c_n D_j}{\mu U_j}, \frac{\tau_n U_j}{D_j}\right) \tag{6}$$

The largest available data set is a collection of all USDA Forest Service data (Skyler and Barry 1990), but other data sets have been published by Bouse and his colleagues, and by Picot and his colleagues. Other researchers (references cited earlier) did not vary test conditions sufficiently, and their data would be difficult to interpret here. The overriding requirement is that enough distinct tests be run to enable the nondimensional parameters to appear in a variety of values to render the definition of fcn in Eq. (6) possible. The number of wind tunnel tests must be large enough to validate and test the constants and coefficients that generate the functional relationships. Suggestions as to the minimum number of tests needed are developed later.

The tests for which unique data are at least possible from the references available (the extensive Spray Drift Task Force data set is not available at this time) are summarized in Table 1. The flat fan and hollow cone nozzles will be examined in detail in the next section, followed by Beecomist and Micronair AU5000 rotary atomizers.

Table 1. Candidate Wind Tunnel Data Sets

NOZZLE TYPE	MATERIAL	NUMBER OF TESTS
From Skyler and Barry (1990):		
Beecomist	Dipel 8L neat	2
	Dipel 8L 1:1	3
	Foray 48B neat	5
	TM Biocontrol	3
	Water	6
Micronair AU5000	Dipel 6L 1:1	3
	Foray 48B neat	11
	Thuricide 32LV 1:1	4
	Water	9
8001	Water	9
8003	Foray 48B neat	3
8004	Foray 48B neat	2
	Water	9
8006	Water	2
8010	Water	11
8020	Water	9
D2-23	Water	6
D2-25	Water	6
D4-45	Water	6
D4-46	Water	6
D8-45	Water	6
D8-46	Water	6
D8 Jet	Water	5
RD-7	Water	6
RD-10	Water	6
From Bouse, Carlton and Jank (1988):		
D6-45	Water with Nalco-Trol	6
	Water with Nalco-Trol II	6
	Water with Sta-Put	6
	Water with Drifgon	6
	Water with 38-F	6
	Water with Wind-Fall	6
D6-46	Water with Nalco-Trol	6
	Water with Nalco-Trol II	6
	Water with Sta-Put	6
	Water with Drifgon	6
	Water with 38-F	6
	Water with Wind-Fall	6
From Picot, van Vliet and Payne (1989):		
Micronair AU5000	Water with Isopropanol	7

6. Water: Flat Fan and Hollow Cone Nozzles

The most obvious first step is to investigate the flow of a Newtonian fluid, water, through the flat fan and hollow cone nozzles. The available data sets (from Skyler and Barry 1990) yield the wind tunnel results summarized in Table 2. Note that the data for the 8006 (Table 1) are not included; we will measure the performance of the technique by comparing its predictions with the 8006 data. Nozzle diameters are obtained from the latest Spraying Systems Company Industrial Spray Products Catalog (No. 51).

An initial examination of the data revealed that much better correlation with data was possible when the velocity present in the Reynolds and Weber numbers is the relative velocity:

$$U_\infty - U_j$$

without a cosine correction for angle β (the cosine correction makes things much worse). Therefore, for the flat fan and hollow cone nozzles, we define:

$$Re = \frac{\rho(U_\infty - U_j)D_j}{\mu} \tag{7}$$

$$We = \frac{\rho(U_\infty - U_j)^2 D_j}{\sigma} \tag{8}$$

The assumed functional relationship is:

$$\frac{D_{0.k}}{D_j} = A_k + B_k \, Re^a \, We^b \tag{9}$$

where $k = 1$, 5, or 9, and the dependence on Re and We is universal across all drop size distributions. This further assumption (the constancy of a and b) permits us to use all three sets of diameters recovered from the wind tunnel tests to develop a least squares algorithm and minimize the standard deviations between the data and the predictions of Eq. (9), while also maximizing the correlation coefficient (Guttman, Wilks and Hunter 1971):

$$R^2 = \frac{\sum (P - \bar{P})(D - \bar{D})}{\sigma_P \, \sigma_D} \tag{10}$$

where D and P are the data and predicted values, respectively, σ_D and σ_P are the standard deviations of the data and predicted values, respectively, and an overbar denotes an average value.

Exploration of all reasonable values of the powers on the Reynolds and Weber numbers suggest that a broad region exists where the correlation coefficient, as computed from Eq. (10), remains fairly constant. A quantitative test could not be constructed to discern the "best" values for a and b. Therefore, it was decided to fit the available data to an $a = -1.0$ power law on Reynolds number. This assumption suggests that agricultural spray atomization behavior is consistent with other atomization applications (Lefebvre 1989) which recover a parametric variation with the Ohnesorge number:

$$Oh = \frac{We^{0.5}}{Re} \tag{11}$$

In addition, it was decided that the power law on Weber number should be the same value irregardless of nozzle orientation. This process leads to the representations:

FLAT FANS AT $\beta = 0$ DEGREES (see Figure 1)

$$\frac{D_{0.1}}{D_j} = 0.0198 + 61.84 \left[\frac{We^{0.42}}{Re}\right] \tag{12}$$

$$\frac{D_{0.5}}{D_j} = 0.1101 + 80.44 \left[\frac{We^{0.42}}{Re}\right] \tag{13}$$

$$\frac{D_{0.9}}{D_j} = 0.2290 + 99.61 \left[\frac{We^{0.42}}{Re}\right] \tag{14}$$

with $R^2 = 0.849$, 0.645 and 0.529 respectively. These results are not as well-correlated as with the other nozzle angles.

FLAT FANS AT $\beta = 90$ DEGREES

$$\frac{D_{0.1}}{D_j} = -0.0217 + 69.70 \left[\frac{We^{0.42}}{Re}\right] \tag{15}$$

$$\frac{D_{0.5}}{D_j} = -0.0318 + 129.50 \left[\frac{We^{0.42}}{Re}\right] \tag{16}$$

$$\frac{D_{0.9}}{D_j} = -0.0186 + 179.98 \left[\frac{We^{0.42}}{Re}\right] \tag{17}$$

with $R^2 = 0.955$, 0.965 and 0.906 respectively.

FLAT FANS AT $\beta = 135$ DEGREES

$$\frac{D_{0.1}}{D_j} = -0.0201 + 65.31 \left[\frac{We^{0.42}}{Re}\right] \tag{18}$$

$$\frac{D_{0.5}}{D_j} = -0.0491 + 129.08 \left[\frac{We^{0.42}}{Re}\right] \tag{19}$$

$$\frac{D_{0.9}}{D_j} = -0.0726 + 189.50 \left[\frac{We^{0.42}}{Re}\right] \tag{20}$$

with $R^2 = 0.934$, 0.978 and 0.974 respectively.

HOLLOW CONES AT $\beta = 0$ DEGREES

$$\frac{D_{0.1}}{D_j} = 0.0143 + 82.12 \left[\frac{We^{0.38}}{Re}\right] \tag{21}$$

$$\frac{D_{0.5}}{D_j} = 0.0352 + 139.00 \left[\frac{We^{0.38}}{Re}\right] \tag{22}$$

$$\frac{D_{0.9}}{D_j} = 0.0731 + 207.20 \left[\frac{We^{0.38}}{Re}\right] \tag{23}$$

with $R^2 = 0.874$, 0.757 and 0.598 respectively. These results are not as well-correlated as with the other nozzle angle.

HOLLOW CONES AT $\beta = 90$ DEGREES

$$\frac{D_{0.1}}{D_j} = -0.0015 + 83.58 \left[\frac{We^{0.38}}{Re}\right] \tag{24}$$

$$\frac{D_{0.5}}{D_j} = 0.0011 + 149.37 \left[\frac{We^{0.38}}{Re}\right] \tag{25}$$

$$\frac{D_{0.9}}{D_j} = 0.0025 + 228.71 \left[\frac{We^{0.38}}{Re}\right] \tag{26}$$

with $R^2 = 0.891$, 0.918 and 0.857 respectively.

These correlations are plotted in Figures 2 - 6.

The above results are not consistent with other researchers who have attempted data compression (Picot, van Vliet and Payne 1989 and Hewitt 1993). Their difficulty was in trying to correlate the effects of non-Newtonian fluids immediately, without first developing the Newtonian effect.

As an example of the accuracy of the above equations, we next predict drop size distribution with the Spraying Systems 8006 nozzle at $\beta = 135$ deg (Table 1), which was not used to generate the constants and coefficients appearing in Eqs. (18) - (20). This example gives:

VARIABLE	PREDICTION	DATA	DIFFERENCE	ERROR
$D_{0.1}$ (µm)	89.29	100.34	11.05	11.0 %
$D_{0.5}$ (µm)	161.69	175.40	13.71	7.8 %
$D_{0.9}$ (µm)	236.52	257.73	21.21	8.2 %

Overall, the accuracy of the curve-fit equations is estimated to be 14 percent.

Table 2a. Wind Tunnel Data for Water: Flat Fan Nozzles

NOZZLE TYPE	ANGLE (deg)	AIR SPEED (m/s)	$D_{0.1}$ (µm)	$D_{0.5}$ (µm)	$D_{0.9}$ (µm)
8001	0	22.4	121.39	204.77	310.56
	0	44.7	121.82	212.10	318.84
	0	67.1	112.53	197.51	284.45
	90	22.4	130.96	232.27	340.15
	90	44.7	93.92	181.56	258.78
	90	67.1	72.54	139.01	196.05
	135	22.4	119.25	218.67	321.39
	135	44.7	97.15	180.54	258.37
	135	67.1	72.68	141.97	194.57
8004	0	22.4	186.29	362.55	549.65
	0	44.7	167.00	333.64	602.21
	0	67.1	136.25	258.51	443.58
	90	22.4	165.50	339.97	528.15
	90	44.7	129.08	242.17	366.23
	90	67.1	97.50	169.72	231.10
	135	22.4	142.66	296.65	452.71
	135	44.7	125.20	201.70	297.58
	135	67.1	90.19	157.55	210.09
8006	45	49.2	155.84	319.84	539.00
	135	53.6	100.34	175.40	257.73
8010	0	22.4	253.22	542.34	1030.60
	0	44.7	215.09	518.31	766.56
	0	67.1	148.24	290.82	571.14
	90	22.4	182.78	413.98	697.76
	90	44.7	133.26	259.51	408.01
	90	60.4	129.71	221.11	317.15
	90	67.1	107.29	182.37	252.18
	135	22.4	159.52	343.93	537.49
	135	44.7	124.43	210.59	316.41
	135	49.2	124.20	212.05	318.42
	135	67.1	103.17	173.49	237.94
8020	0	22.4	300.60	686.11	1001.48
	0	44.7	235.81	609.67	996.89
	0	67.1	148.74	332.18	620.58
	90	22.4	187.27	478.49	797.44
	90	44.7	146.03	294.51	483.75
	90	67.1	124.12	211.80	425.30
	135	22.4	179.58	394.48	616.75
	135	44.7	132.19	236.83	358.26
	135	67.1	113.74	184.56	262.42

Table 2b. Wind Tunnel Data for Water: Hollow Cone Nozzles

NOZZLE TYPE	ANGLE (deg)	AIR SPEED (m/s)	$D_{0.1}$ (µm)	$D_{0.5}$ (µm)	$D_{0.9}$ (µm)
D2-23	0	22.4	123.31	208.15	342.88
	0	44.7	103.75	186.56	281.31
	0	67.1	110.34	171.51	245.12
	90	22.4	106.85	195.90	297.32
	90	44.7	109.32	193.10	278.70
	90	67.1	87.39	151.18	208.00
D2-25	0	22.4	128.39	219.76	348.85
	0	44.7	120.57	198.02	304.55
	0	67.1	114.64	188.74	267.60
	90	22.4	120.85	211.48	330.16
	90	44.7	122.42	196.66	283.67
	90	67.1	81.74	155.96	226.72
D4-45	0	22.4	162.42	291.65	466.90
	0	44.7	141.41	254.56	413.67
	0	67.1	132.50	214.09	302.95
	90	22.4	142.40	263.62	407.78
	90	44.7	131.17	223.02	332.65
	90	67.1	92.57	171.22	252.25
D4-46	0	22.4	208.59	437.43	731.70
	0	44.7	173.78	348.51	598.94
	0	67.1	144.12	265.50	437.32
	90	22.4	171.57	349.61	584.26
	90	44.7	133.90	250.48	399.91
	90	67.1	97.06	175.72	256.35
D8-45	0	22.4	193.34	384.47	601.39
	0	44.7	164.45	320.28	511.65
	0	67.1	138.27	239.75	404.31
	90	22.4	164.60	348.15	544.08
	90	44.7	140.79	261.32	443.21
	90	67.1	110.07	185.67	267.00
D8-46	0	22.4	245.23	501.48	902.93
	0	44.7	207.66	454.75	896.87
	0	67.1	153.97	270.67	474.35
	90	22.4	198.20	442.35	781.92
	90	44.7	159.03	303.87	511.76
	90	67.1	108.52	189.29	274.57

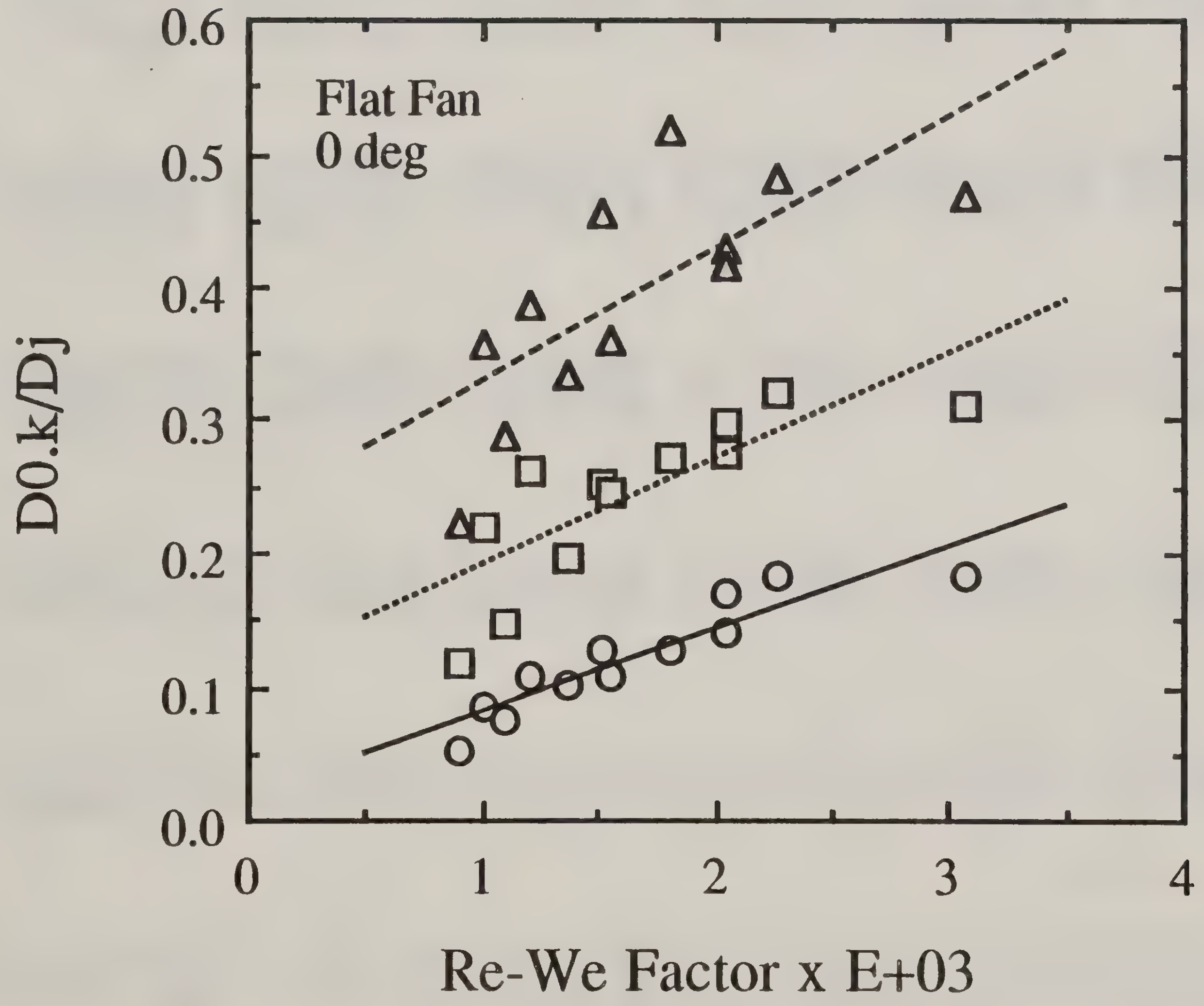

Figure 2. Correlation of wind tunnel data with the curve-fit equations for flat fan nozzles at 0 deg to the air stream. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.42} / Re$.

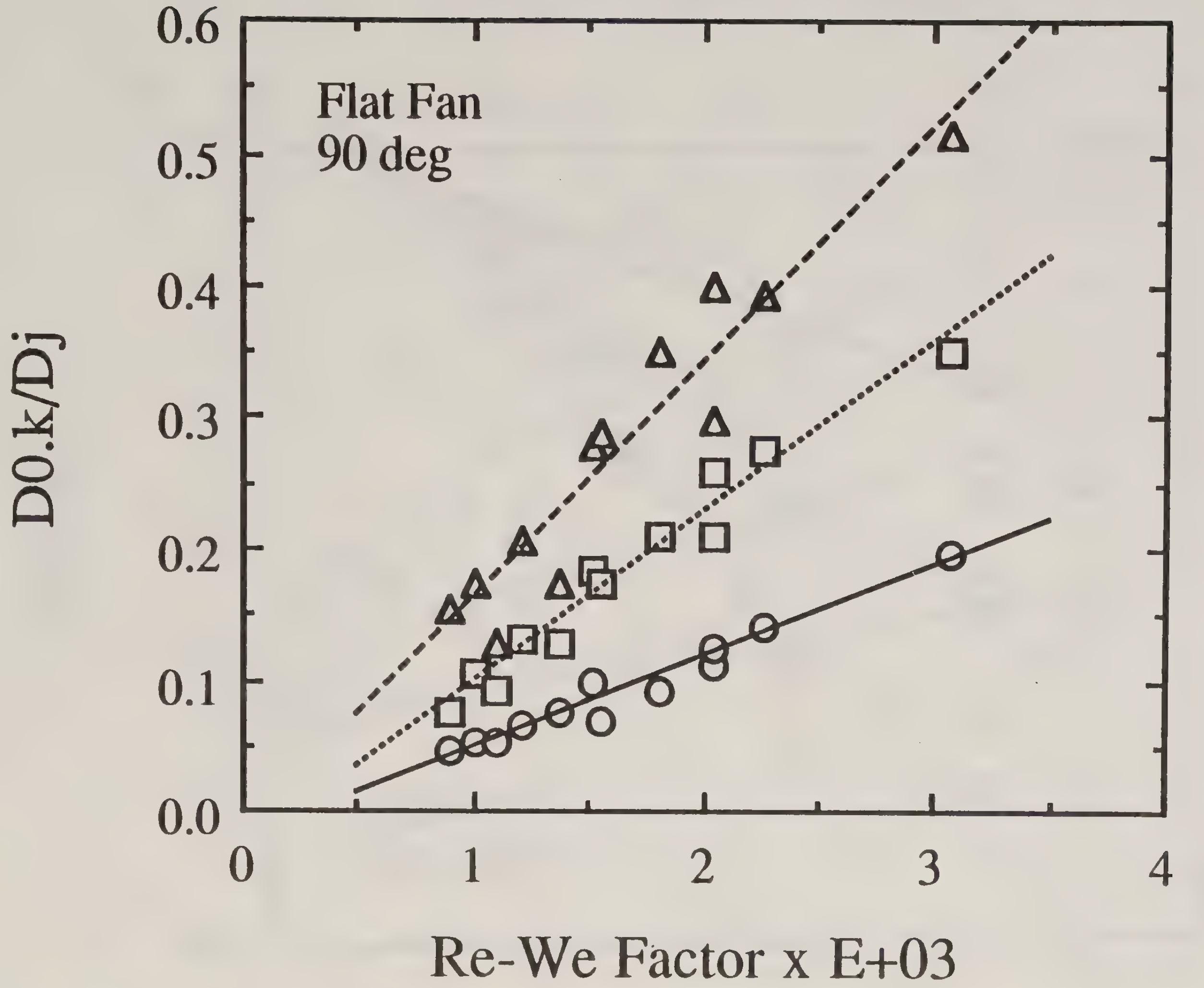

Figure 3. Correlation of wind tunnel data with the curve-fit equations for flat fan nozzles at 90 deg to the air stream. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.42} / Re$.

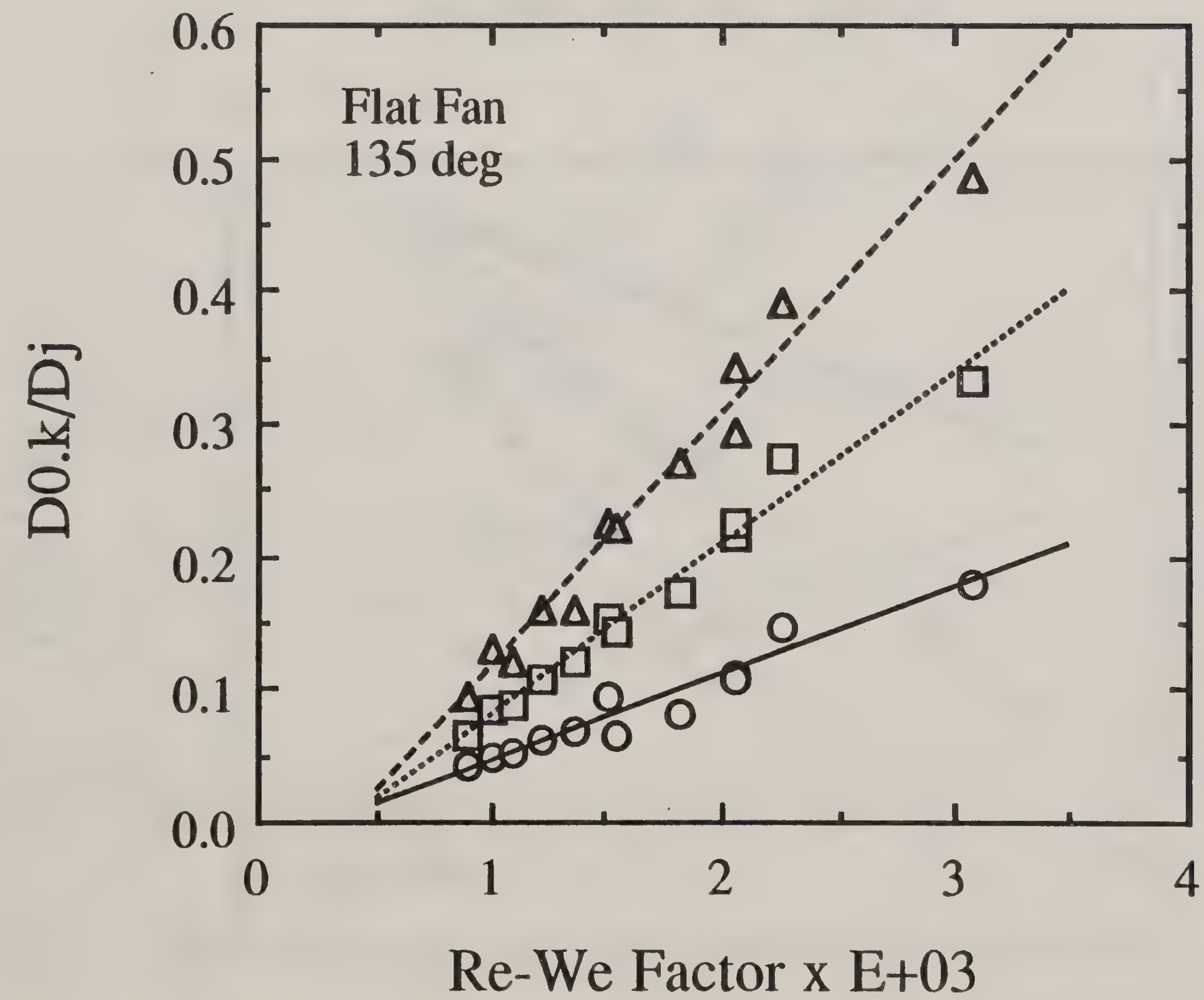

Figure 4. Correlation of wind tunnel data with the curve-fit equations for flat fan nozzles at 135 deg to the air stream. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.42}$ / Re.

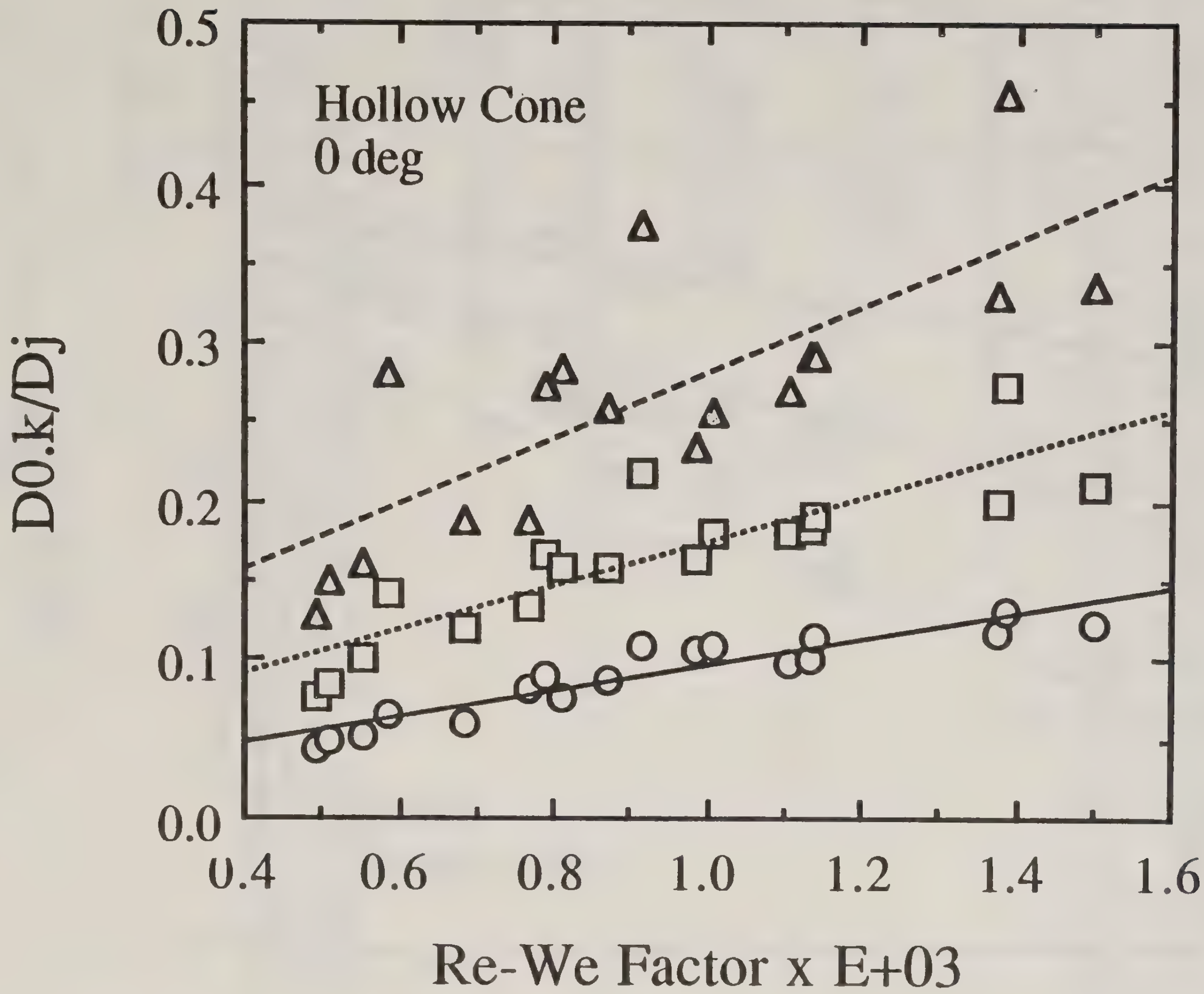

Figure 5. Correlation of wind tunnel data with the curve-fit equations for hollow cone nozzles at 0 deg to the air stream. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.38} / Re$.

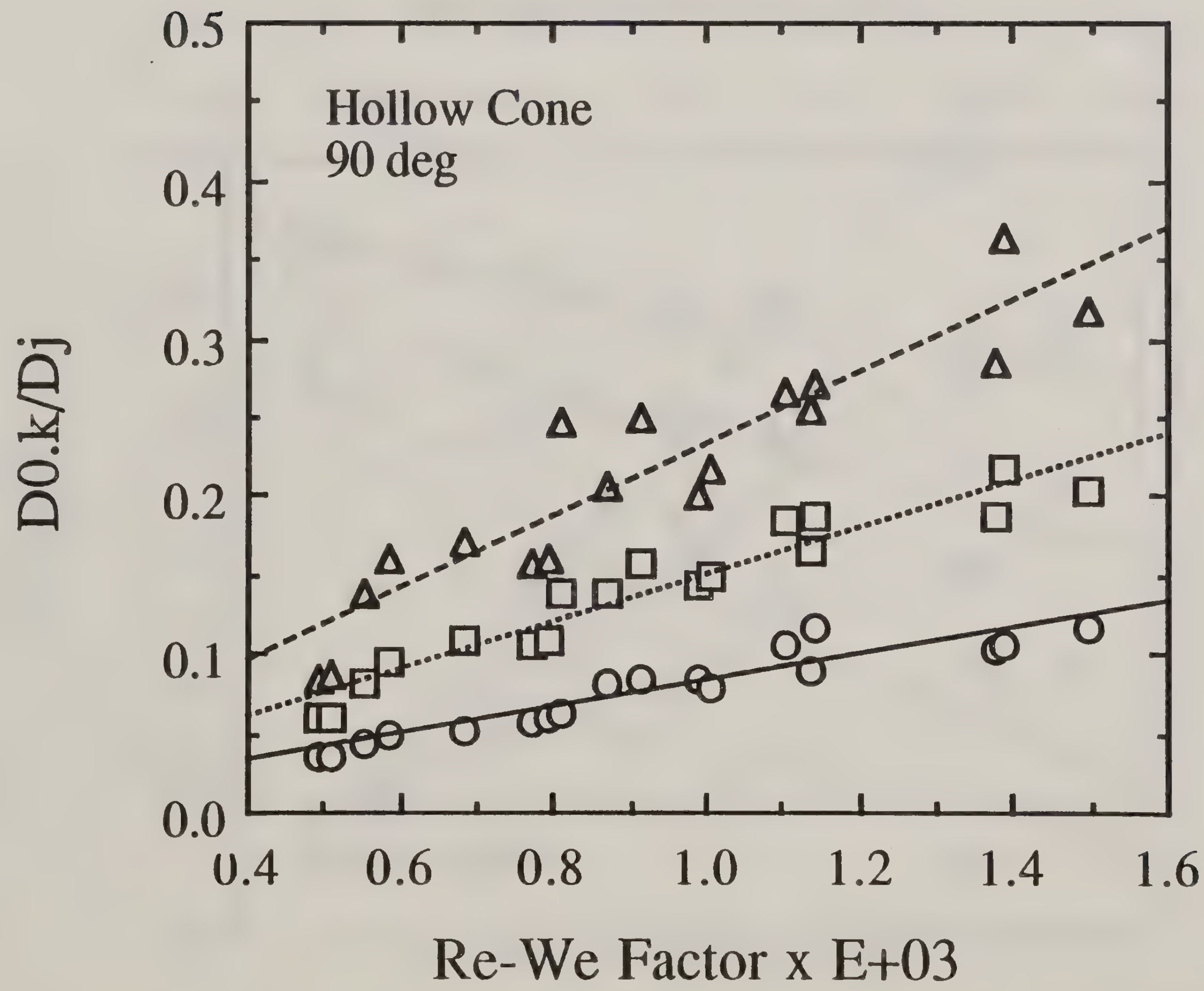

Figure 6. Correlation of wind tunnel data with the curve-fit equations for hollow cone nozzles at 90 deg to the air stream. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.38} / Re$.

7. Water: Rotary Atomizers

The next step is to investigate the flow of a Newtonian fluid, water, through rotary atomizers. Again, the available data sets (from Skyler and Barry 1990) yield the wind tunnel results summarized in Table 3. Note that the data contains both Micronair and Beecomist atomizers. Details of the Micronair AU5000 are obtained from Picot, van Vliet and Payne (1989), and for the Beecomist, from P. G. Phillips (private communication).

Both Picot, van Vliet and Payne (1989) and Parkin and Siddiqui (1990) correlated rotary atomizer data and found their best results when the characteristic size of the atomizer is taken to be D_c, the diameter of the cylindrical mesh shell. Also, Parkin and Siddiqui (1990) found that the rotation rate best correlates with their data as the inverse square root. This information will be used here to extend the results presented above to rotary atomizer behavior.

Therefore, for rotary atomizers, we define:

$$Re = \frac{\rho(U_\infty - U_j)D_c}{\mu} \tag{27}$$

$$We = \frac{\rho(U_\infty - U_j)^2 D_c}{\sigma} \tag{28}$$

The assumed functional relationship is:

$$\frac{D_{0.k}}{D_c} = A_k + B_k\, Re^a\, We^b / \Pi_r{}^{0.5} \tag{29}$$

where $k = 1$, 5, or 9, and the dependence on Re and We is again universal across all drop size distributions and rotary atomizer types. This assumption (the constancy of a and b) permits us to use all three sets of diameters recovered from the wind tunnel tests to develop a least squares algorithm and minimize the standard deviations between the data and the predictions. This process leads to the representations:

ROTARY ATOMIZERS

$$\frac{D_{0.1}}{D_c} = -0.00083 + 303.57 \left[\frac{We^{0.23}}{Re\, \Pi_r^{0.5}}\right] \tag{30}$$

$$\frac{D_{0.5}}{D_c} = -0.00122 + 509.60 \left[\frac{We^{0.23}}{Re\, \Pi_r^{0.5}}\right] \tag{31}$$

$$\frac{D_{0.9}}{D_c} = -0.00167 + 721.03 \left[\frac{We^{0.23}}{Re\, \Pi_r^{0.5}}\right] \tag{32}$$

with $R^2 = 0.813$, 0.897 and 0.861 respectively. The correlation is plotted in Figure 7.

Parkin and Siddiqui (1990) apply a least-squares algorithm to their data to arrive at the power-law dependences:

$U_\infty^{0.18}$

$D_c^{-0.70}$

while the above results give:

$(U_\infty - U_j)^{-0.04}$

$D_c^{-1.27}$

Our analysis suggests an attempt by the nondimensional parameters to "remove" air speed as a variable in rotary atomizers.

Table 3. Wind Tunnel Data for Water: Rotary Atomizers

NOZZLE TYPE	ROTATION (rpm)	AIR SPEED (m/s)	$D_{0.1}$ (μm)	$D_{0.5}$ (μm)	$D_{0.9}$ (μm)
AU5000	3750	22.4	106.81	248.73	284.02
	9100	44.7	56.36	110.17	162.12
	11700	58.1	49.01	81.37	138.52
	4100	22.4	142.01	306.49	404.20
	8000	44.7	56.35	118.19	179.08
	10850	58.1	44.08	79.13	131.12
	4200	44.7	96.31	200.18	278.70
	7000	44.7	104.84	162.16	219.84
	11000	60.3	52.64	110.54	174.23
Beecomist	11700	22.4	55.97	124.82	173.30
	11700	44.7	59.65	123.46	170.42
	8300	22.4	111.13	191.05	285.92
	8350	44.7	112.65	178.25	270.46
	9000	24.6	128.27	187.85	282.84
	7000	24.6	147.52	248.95	382.27

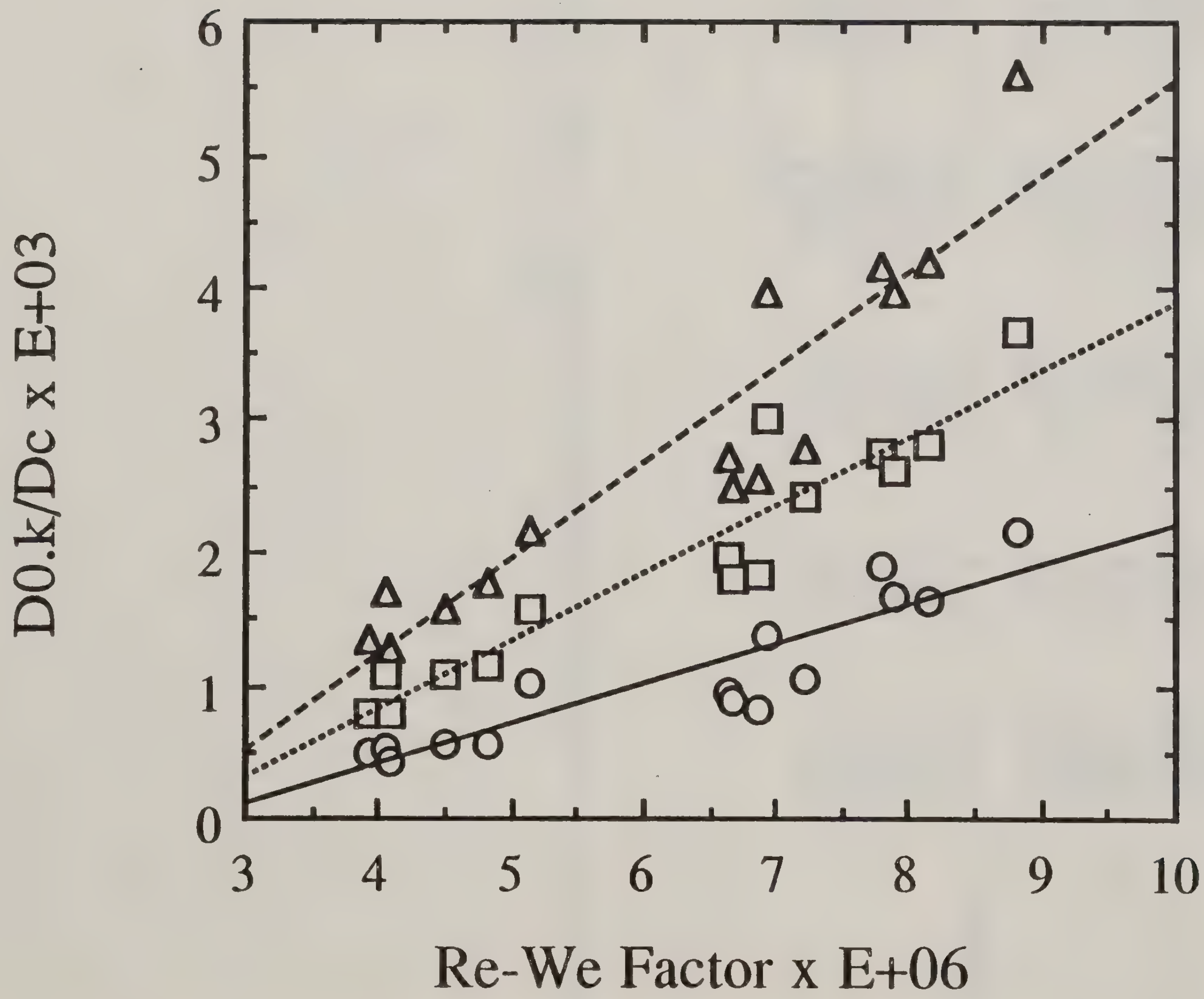

Figure 7. Correlation of wind tunnel data with the curve-fit equations for rotary atomizers. $D_{0.1}$ is represented by circles (data) and the solid line (curve fit); $D_{0.5}$ by squares (data) and the dotted line (curve fit); and $D_{0.9}$ by triangles (data) and the dashed line (curve fit). The Re-We Factor is $We^{0.23} / Re\ \Pi_r^{0.5}$.

8. Non-Newtonian Fluids

The addition of viscoelastic effects requires us to expand the functional relationship of Eq. (9) to read:

$$\frac{D_{0.k}}{D_j} = [\, A_k + B_k \, Re^a \, We^b \,][\, 1 + C_k \, \Pi_s{}^c \,][\, 1 + D_k \, De^d \,] \tag{33}$$

where A_k, B_k, a and b are defined previously (these constants and coefficients should not change when fluids other than water are used; the rotation rate behavior is also added for rotary atomizers). Because of the nature of non-Newtonian fluids, we expect C_k and D_k to be positive numbers. We identify the two non-Newtonian parameters by:

$$\Pi_s = \frac{c_n D_j}{\mu(U_\infty - U_j)} \tag{34}$$

$$De = \frac{\tau_n (U_\infty - U_j)}{D_j} \tag{35}$$

An alternate expression for the non-Newtonian terms could be:

$$[\, 1 + E_k \, \Pi_s{}^c \, De^d \,] \tag{36}$$

combining the individual expansions. Sufficient wind tunnel data will be needed to detail which terms to use here.

The existing data bases do not include enough data to define quantitatively the new constants and coefficients in Eq. (33) above. Two difficulties arise when attempting to make use of the existing data sets:

1. The viscoelastic fluid parameters c_n and τ_n are not known, or have not been determined for any spray material in the data base. However, these constants could be worked into modifications of the constants C_k and D_k, with the use of the data sets to "back out" their values. Unfortunately, this then leads to:

2. There are too few unique wind tunnel tests to close the problem. For the least squares technique to work with some degree of confidence, there should be at least twice as many data points as unknown constants or coefficients. In the case of viscoelastic fluids through flat fan and hollow cone nozzles, even with c_n and τ_n known, four constants must be evaluated, requiring eight wind tunnel tests. With the viscoelastic

physical properties not known from separate benchtop tests, an additional constant must be determined, requiring two additional wind tunnel tests.

Physical properties of the spray material must be determined, then additional wind tunnel tests must be performed, before we can quantify the behavior of non-Newtonian fluids in this application.

Picot, van Vliet and Payne (1989) contains seven tests with the Micronair AU5000 for a non-Newtonian fluid, water with nine percent isopropanol (Table 4). This paper also includes measurements of the surface tension and viscosity. If we use the alternate expression (Eq. (36) above) and expand the non-Newtonian terms, we obtain the expression:

$$\left[1 + F_k \left(\frac{D_c}{\mu\,(U_\infty - U_j)} \right)^c \left(\frac{U_\infty - U_j}{D_c} \right)^d \right] \tag{37}$$

with:

$$F_k = E_k\, c_n{}^c\, \tau_n{}^d \tag{38}$$

This approach contains three constants: the powers c and d and the coefficient F_k. However, all of the data is collected at the same air speed, which suggests that the terms in parentheses in Eq. (37) will change very little in a least-squares analysis. This effect further suggests that the best possible examination of this data would be to assume that the non-Newtonian terms may be approximated by the behavior:

$$[\, 1 + G_k \,] \tag{39}$$

essentially a constant multiplying the Newtonian result. This approach leads to the values of:

$$\begin{aligned} G_k &= 5.85 \quad \text{for } D_{0.1} \\ G_k &= 3.90 \quad \text{for } D_{0.5} \\ G_k &= 3.39 \quad \text{for } D_{0.9} \end{aligned} \tag{40}$$

with $R^2 = 0.754$, 0.834, and 0.788 respectively. These results are encouraging, but indicate the need for further study and laboratory or wind tunnel data. Equation (40) would suggest that the non-Newtonian effect is fairly significant.

Table 4. Wind Tunnel Data for Water/Isopropanol: Rotary Atomizers

NOZZLE TYPE	ROTATION (rpm)	AIR SPEED (m/s)	$D_{0.1}$ (μm)	$D_{0.5}$ (μm)	$D_{0.9}$ (μm)
AU5000	7920	44.7	42.2	106.0	182.0
	11000	44.7	41.1	94.0	175.0
	3660	44.7	57.7	165.0	261.0
	7920	44.7	43.9	126.0	216.0
	11000	44.7	47.8	115.0	195.0
	3660	44.7	71.7	161.0	284.0
	7920	44.7	49.2	136.0	239.0

9. Drop Size Distributions

With the $D_{0.1}$, $D_{0.5}$, and $D_{0.9}$ drop diameters in hand, we may then turn to the construction of the entire drop size distribution profile, to obtain an indication of the accuracy we have implied in the dimensional analysis approach. As an example here, we will use the 8006 flat fan nozzle result (at an exit angle of $\beta = 135$ deg), as given in the Skyler and Barry (1990) data base (and plotted in Figure 8), whose cumulative volume fraction CVF is plotted in Figure 9. For this example, Table 2 indicates that:

$$D_{0.1} = 100.34\ \mu m$$
$$D_{0.5} = 175.40\ \mu m$$
$$D_{0.9} = 257.73\ \mu m$$

Several techniques have been developed to curve fit the wind tunnel drop size distribution. The most widely used expression for displaying drop size distribution is the logarithmic normal representation (Rosin and Rammler 1933):

$$1 - CVF = \exp\left[-(D / X)^q \right] \qquad (41)$$

where X and q are constants, and D is the drop diameter. Traditionally, X is taken as the drop diameter $D_{0.632}$, and q is the slope of the least squares curve fit through the data in transform space. For the 8006 data, transform space may be plotted as shown in Figure 10 (with the parameters $D_{0.632} = 198.47\ \mu m$ and $q = 3.1316$). The resulting drop size distribution comparison is given in Figure 11 ($R^2 = 0.978$), and is seen to miss at the maximum volume fractions (near $D_{0.5}$) and be high at the larger drop diameters.

A more complicated technique is the upper-limit function (Mugele and Evans 1957), requiring the determination of three constants correlating CVF to an error function behavior. Goering and Smith (1978) give approximate formula for generating these three parameters:

$$D_m = \frac{D_{0.5} (D_{0.1} + D_{0.9}) - 2\, D_{0.1}\, D_{0.9}}{D_{0.5} - D_{0.1}\, D_{0.9} / D_{0.5}} \qquad (42)$$

$$A = \frac{D_m - D_{0.5}}{D_{0.5}} \qquad (43)$$

$$\delta = \frac{0.394}{\log_{10} \left(\dfrac{A\, D_{0.9}}{D_m - D_{0.9}} \right)} \qquad (44)$$

to give

$$CVF = 0.5\ (\ 1 + \text{erf}\ (\ \delta\ z\ )\ ) \tag{45}$$

$$z = \log\ (\ \frac{A\ D_i}{D_m - D_i}\ ) \tag{46}$$

and erf is the commonly-defined error function. For the 8006 data, $D_m = 396.40\ \mu m$, $A = 1.260$, and $\delta = 1.066$. The correlation to CVF is shown in Figure 12, and the drop size distribution comparison is given in Figure 13 ($R^2 = 0.987$). The curvefit is closer to the data than the Rosin-Rammler representation of Figure 11. This approach has recently been extended to four constants for better accuracy (Patel and Gaidos 1991).

The technique preferred here (Teske, Skyler and Barry 1991; Teske and Barry 1992; and Teske 1992) was developed by Simmons (1977) from an idea he attributed to Tate and Marshall (1953). In this technique CVF is plotted on a normal probability distribution scale as a function of the square root of the drop diameter. The results were generally so startling that the authors felt compelled to dismiss divine intervention and intimate that the correlation was purely empirical and should not be used as a basis for theoretical interpretation.

If the drop diameters are normalized by $D_{0.5}$, and a least-squares fit is invoked in root/normal space, with an iteration technique employed to maximize the area under the volume fraction (Esterly 1991), Figures 14 and 15 ($R^2 = 0.989$) result. In this case only two parameters are needed to describe the distribution: $D_{0.5}$ and the slope a of the straight line in Figure 14. Here $D_{0.5} = 176.47\ \mu m$ and $a = 0.1738$, and only the tail of the distribution is predicted incorrectly.

If we take the 8006 data and realize that:

$$a = \frac{\sqrt{\frac{D_{0.9}}{D_{0.5}}} - \sqrt{\frac{D_{0.1}}{D_{0.5}}}}{Pr(0.9) - Pr(0.1)} \tag{47}$$

where Pr( ) is the normal probability distribution function (forwards and backwards in equations 26.2.18 and 26.2.23, respectively, of Abramowitz and Stegun 1968), for which $Pr(0.9) = -Pr(0.1) = 1.282$. In this application the slope parameter becomes $a = 0.1778$, an especially close fit to the slope in Figure 14.

Alternately, the relative span RS, defined by:

$$RS = \frac{D_{0.9} - D_{0.1}}{D_{0.5}} \tag{48}$$

may be supplied. Through manipulation of the root/normal function, Eq. (47) may be replaced by the solution to the equation:

$$a = \frac{RS}{4\,Pr(0.9)} = 0.195\ RS \tag{49}$$

For this case, $RS = 0.897$ from Eq. (48), and $a = 0.1749$ from Eq. (49), an even better fit to the slope in Figure 14.

If we use the root/normal approach on the predicted diameter values (through Eqs. (18) - (20)):

$D_{0.1} =$ 89.29 µm

$D_{0.5} =$ 161.69 µm

$D_{0.9} =$ 236.52 µm

which give $a = 0.1776$ from Eq. (49), the predicted drop size distribution that results is shown in Figure 16 ($R^2 = 0.932$). Here it may be seen that the prediction closely matches the actual wind tunnel data.

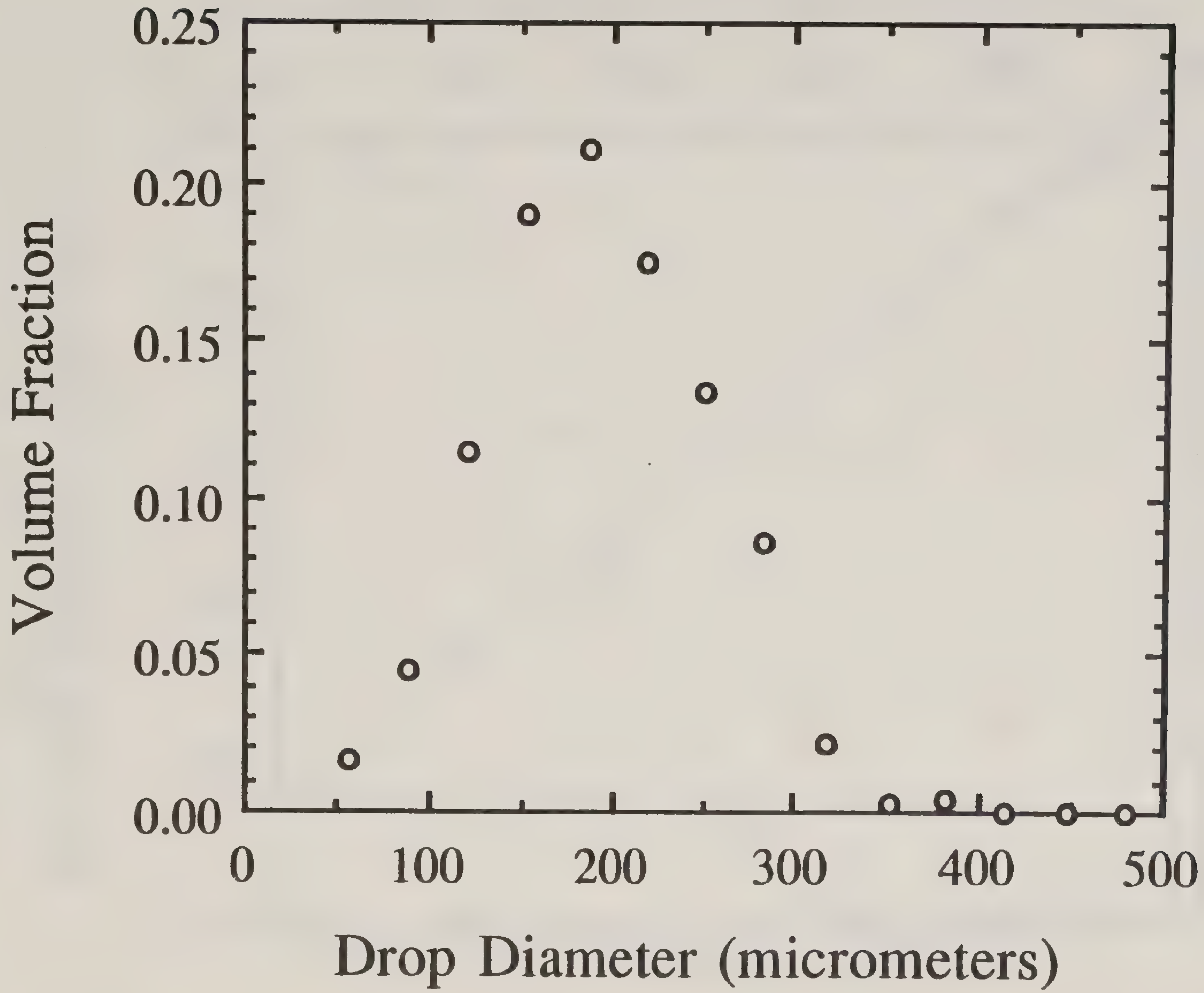

Figure 8. Drop size distribution volume fraction for an 8006 flat fan nozzle, spraying water and oriented 135 deg to a 53.6 m/s air stream. Pertinent drop diameters are $D_{0.1}$ = 100.34 μm ; $D_{0.5}$ = 175.40 μm ; and $D_{0.9}$ = 257.73 μm (from the data in Skyler and Barry 1990).

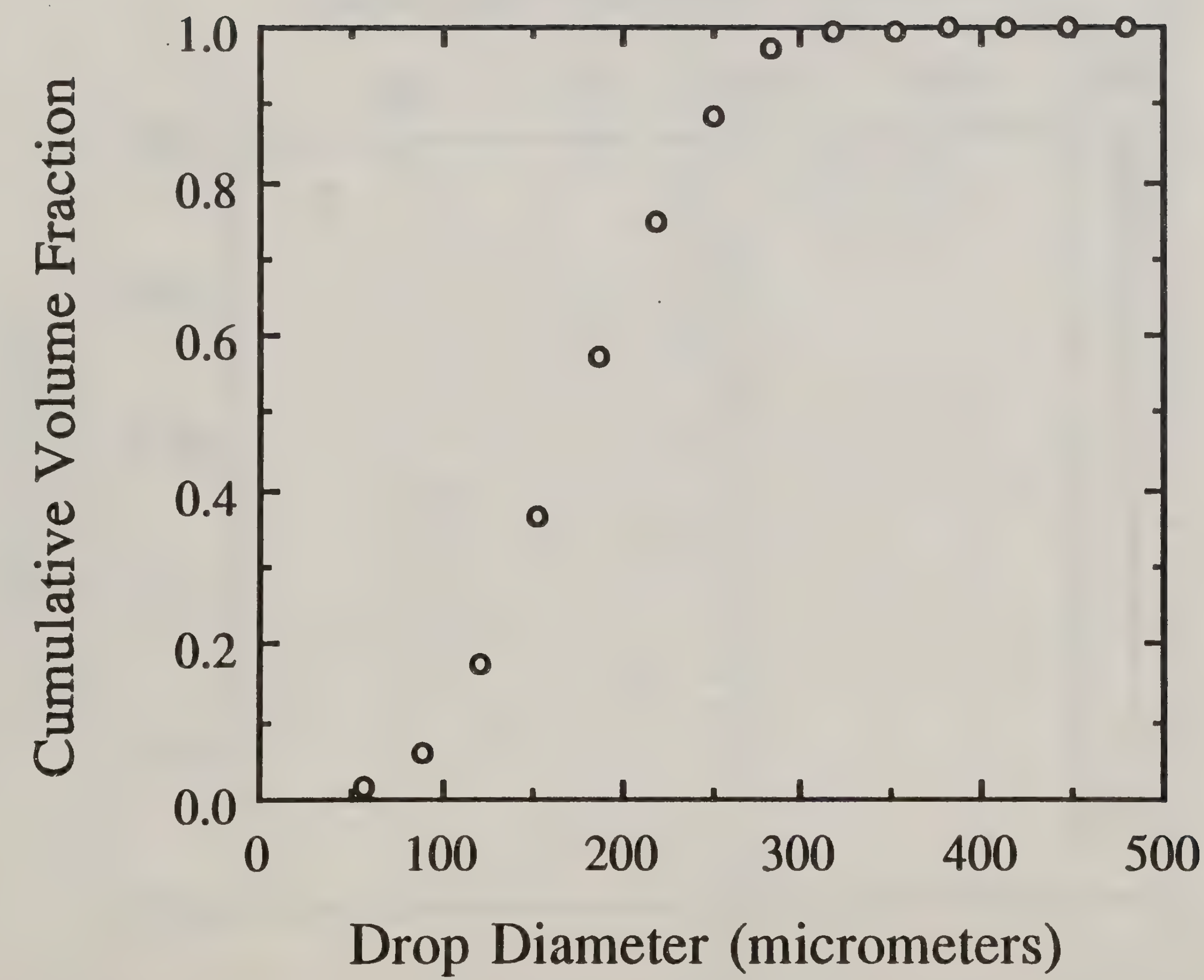

Figure 9. Cumulative volume fraction for the drop size distribution given in Figure 8.

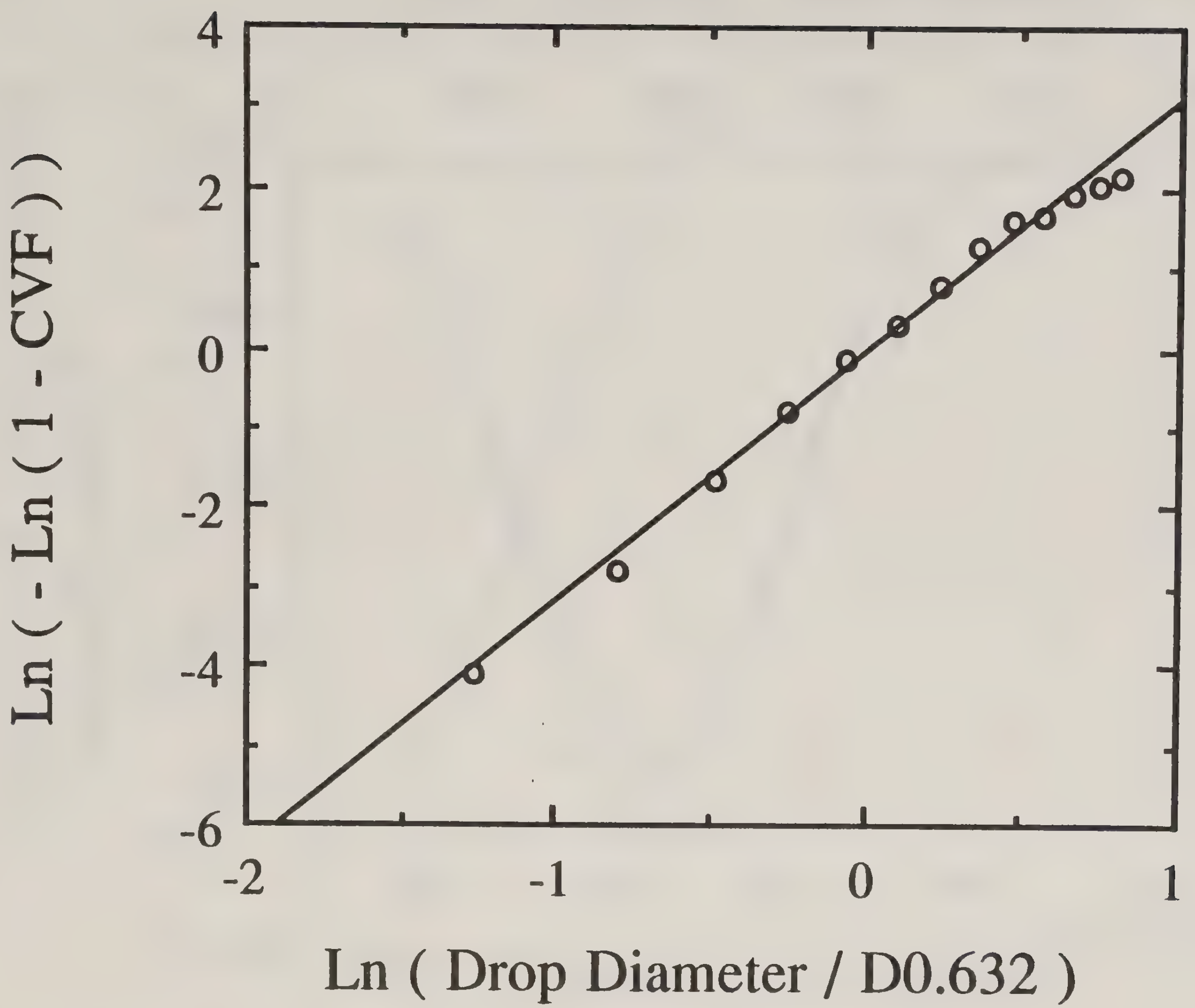

Figure 10. Rosin-Rammler logarithmic normal representation of the drop size distribution given in Figure 8 (circles). In this case $D_{0.632} = 198.47\ \mu m$ (from the data in Skyler and Barry 1990), and a least-squares fit gives the value of the slope $q = 3.1316$.

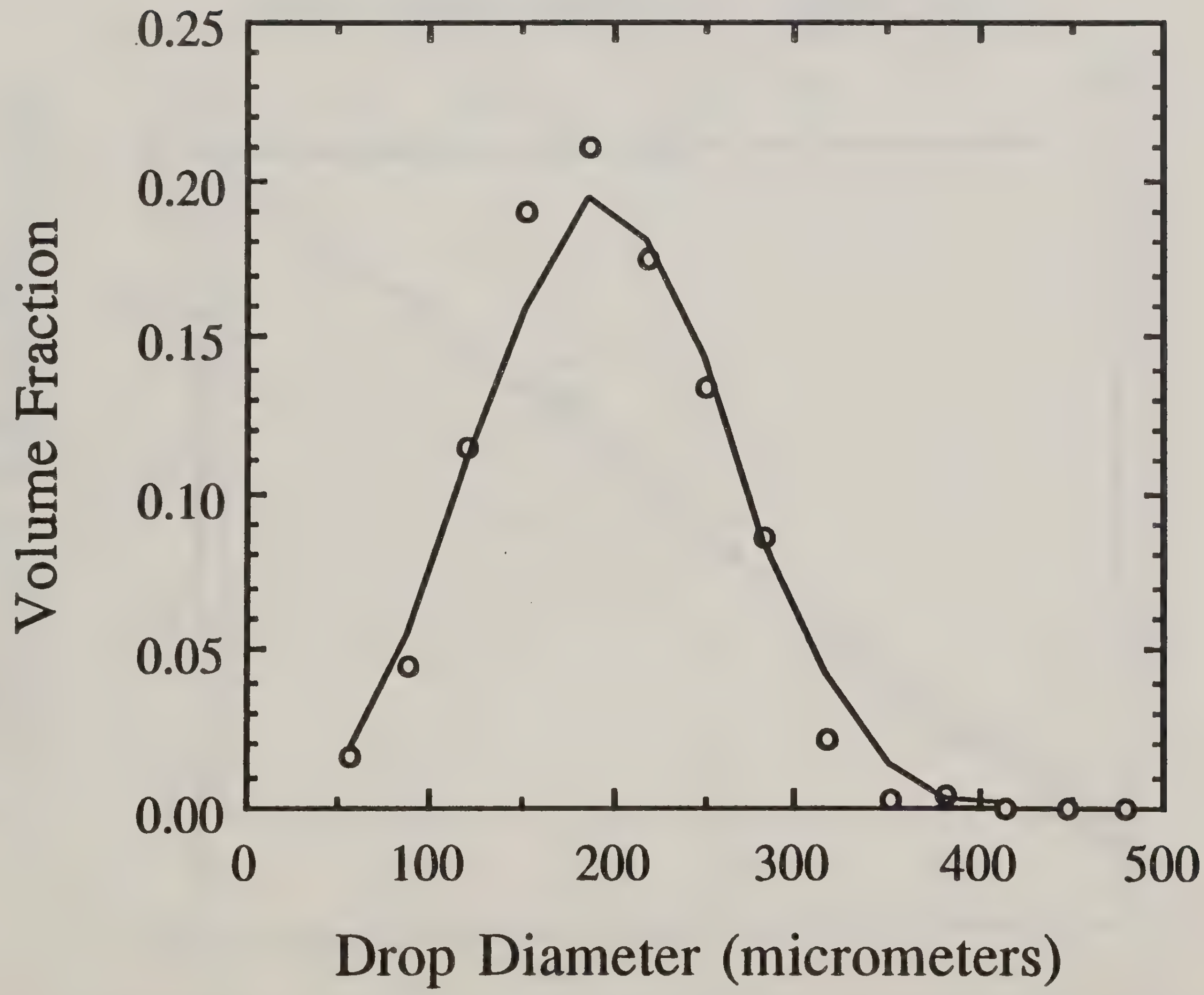

Figure 11. Reconstruction of the drop size distribution given in Figure 8 (circles) with the Rosin-Rammler logarithmic normal representation in Figure 10.

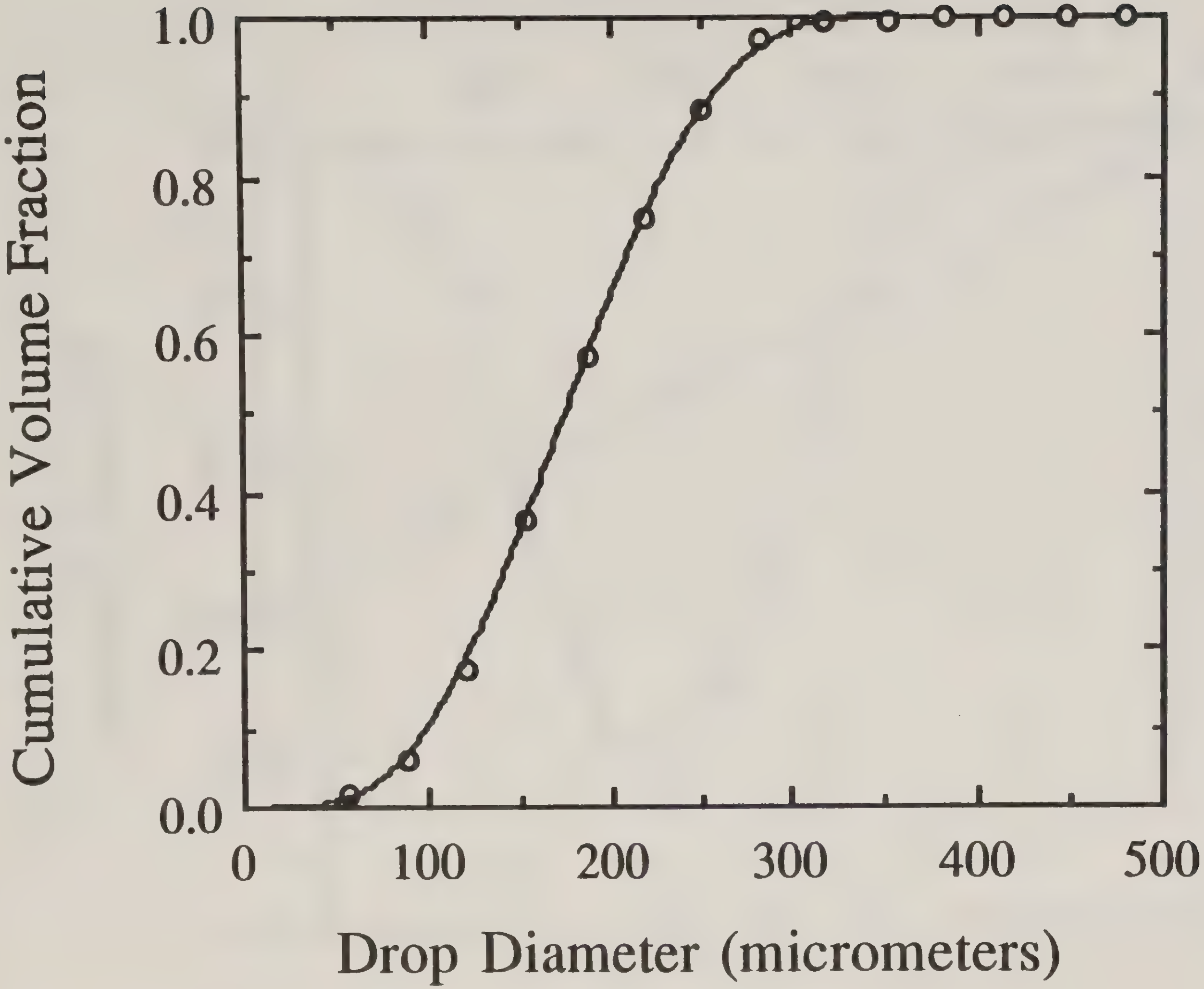

Figure 12. Mugele-Evans upper-limit function representation of the drop size distribution given in Figure 8 (circles). The pertinent parameters are $D_m = 396.40\ \mu m$; $A = 1.260$; and $\delta = 1..066$.

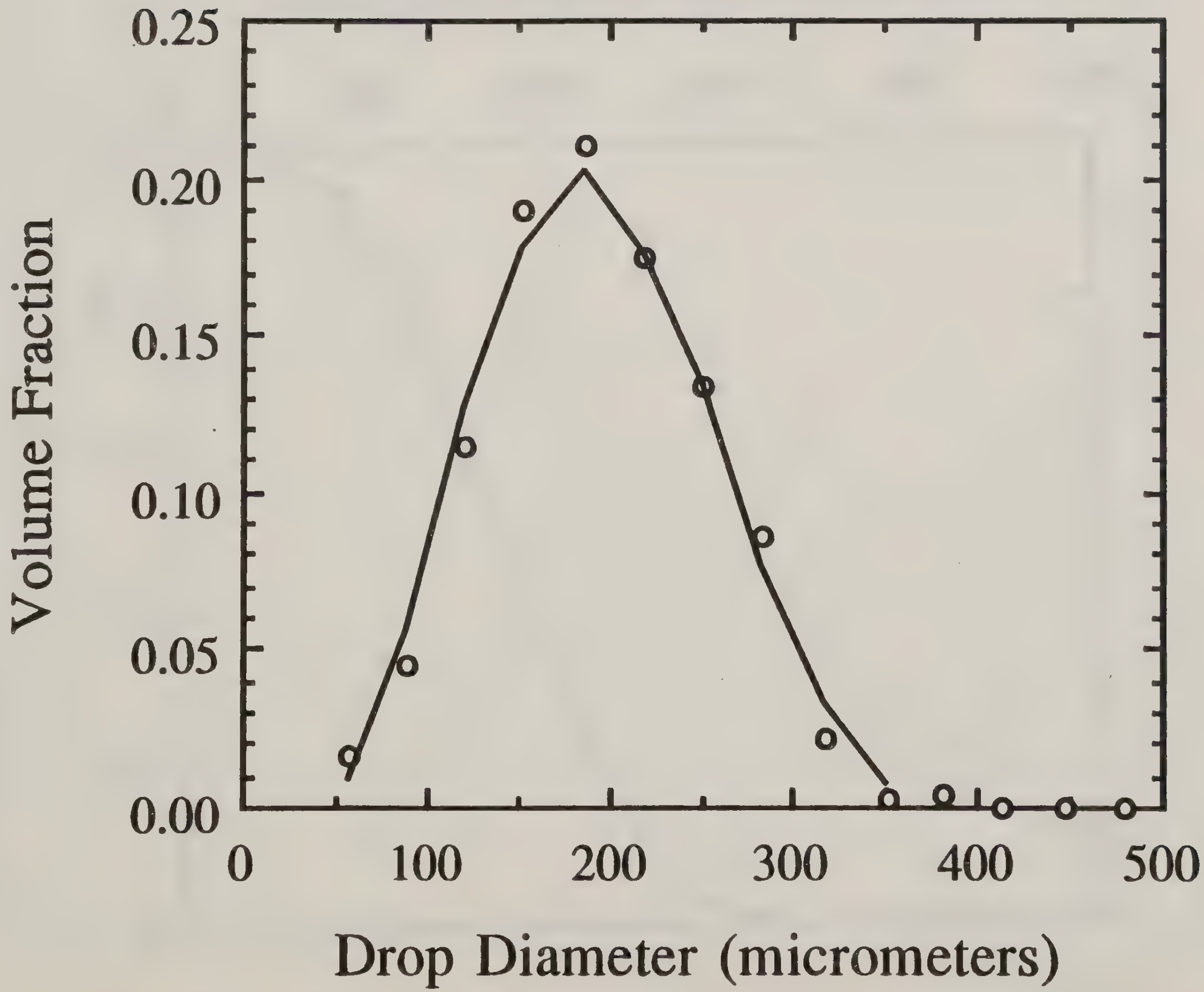

Figure 13. Reconstruction of the drop size distribution given in Figure 8 (circles) with the Mugele-Evans upper-limit function in Figure 12.

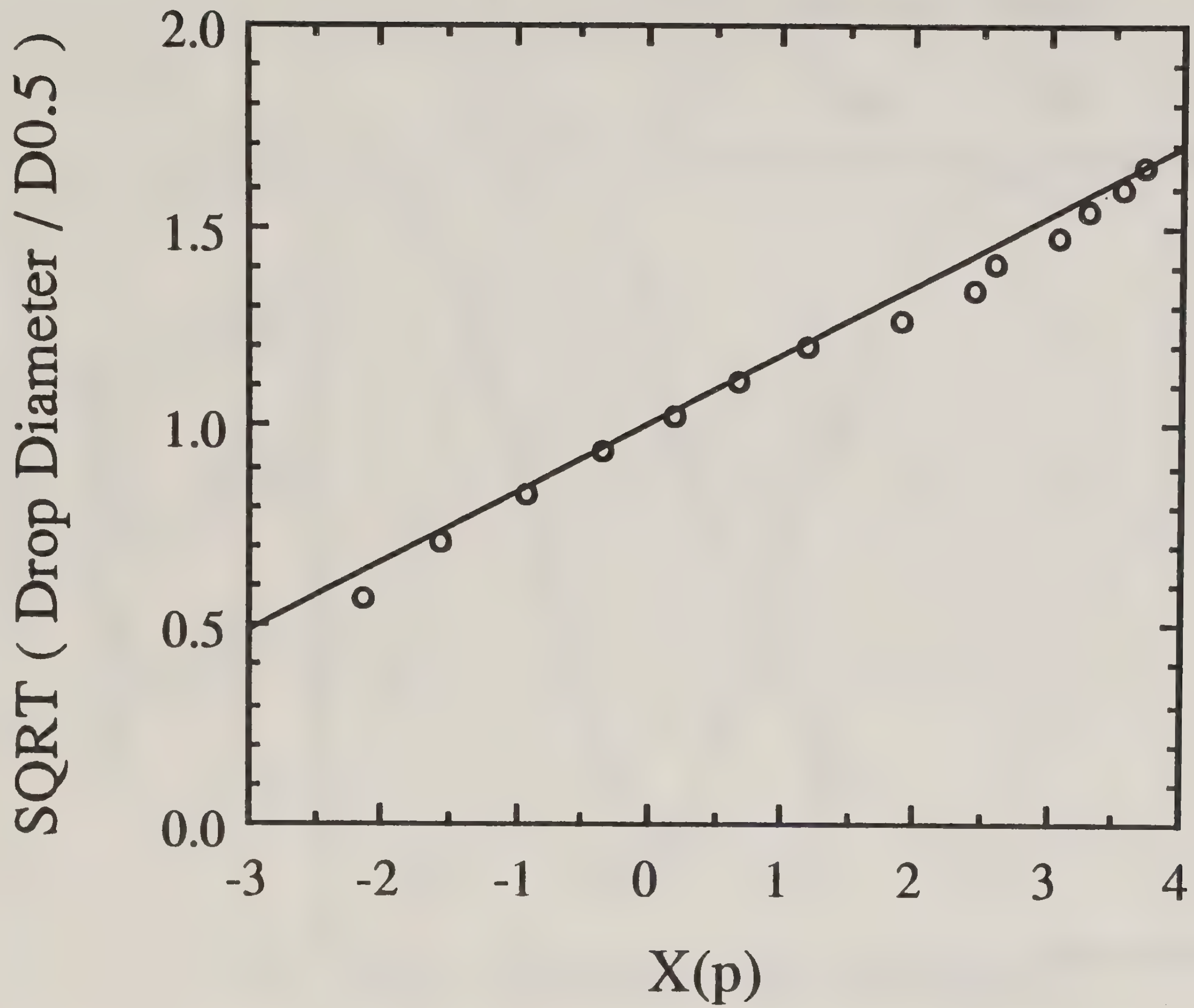

Figure 14. Root/normal probability representation of the drop size distribution given in Figure 8 (circles), plotted against the square root of the drop diameter. X(p) is such that at X(p) = 0 , the probability equals 0.5 .

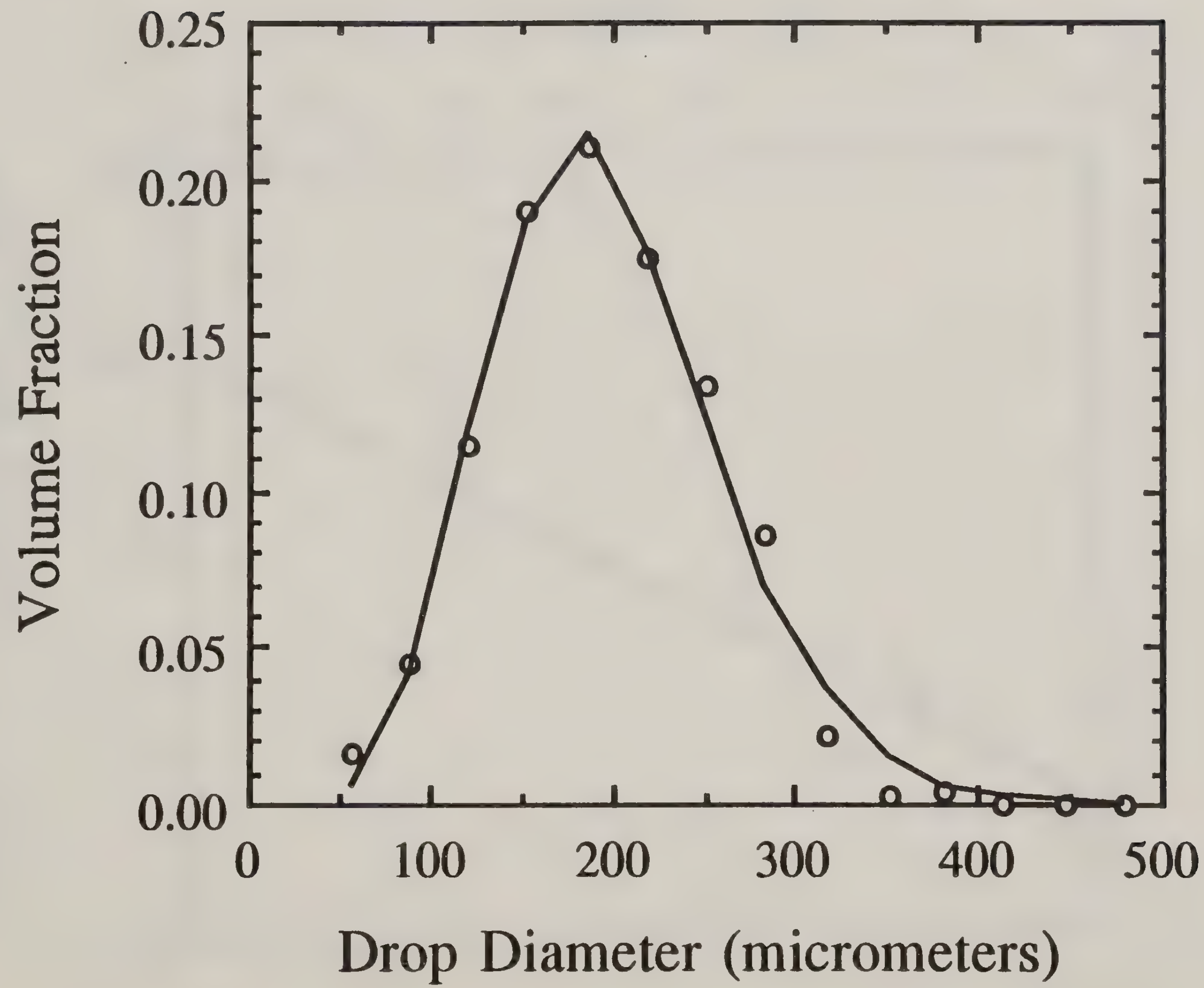

Figure 15. Reconstruction of the drop size distribution given in Figure 8 (circles) with the root/normal probability representation in Figure 14.

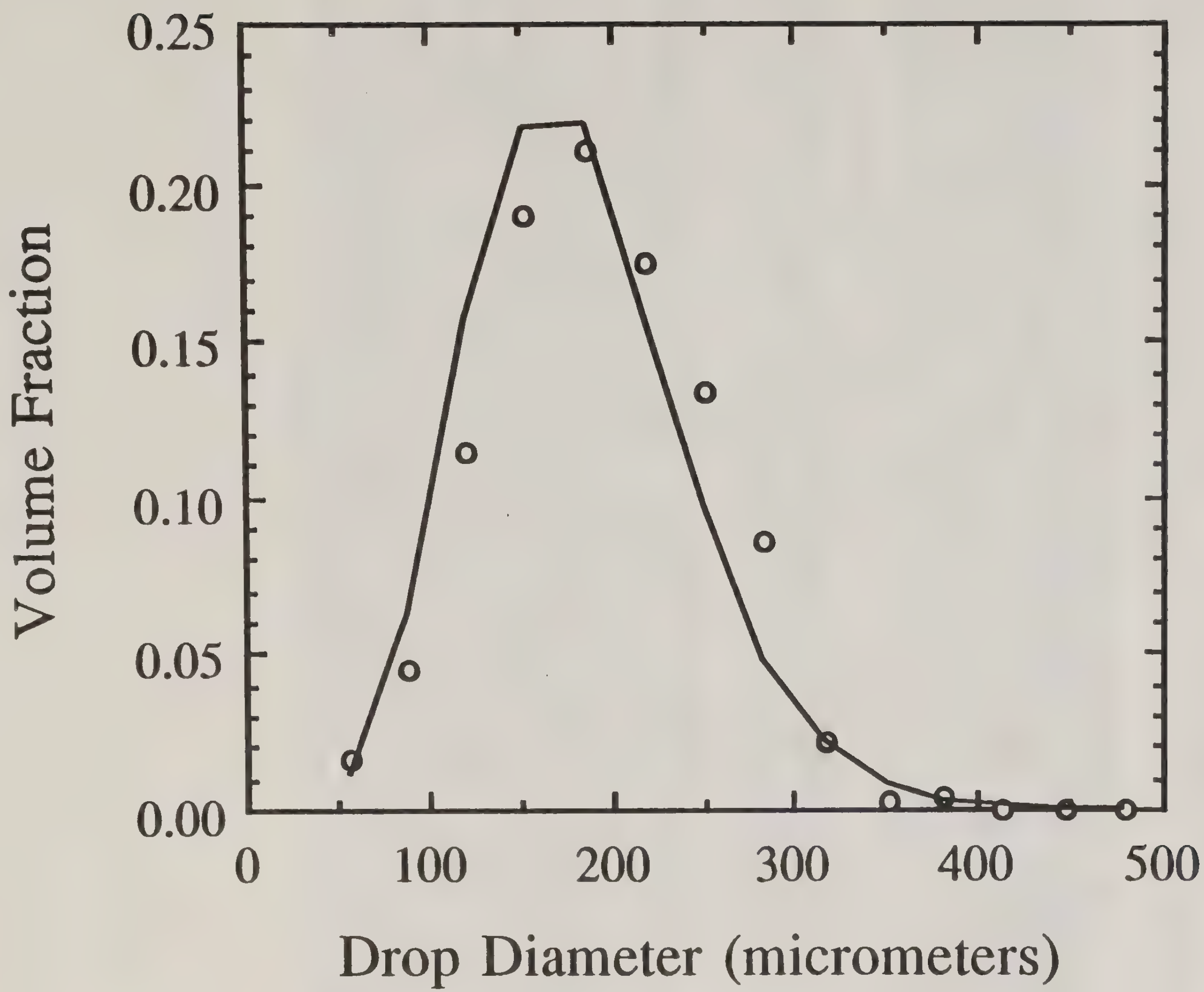

Figure 16. Reconstruction of the drop size distribution given in Figure 8 (circles) with the root/normal probability representation applied to the curve-fit predictions of $D_{0.1}$, $D_{0.5}$ and $D_{0.9}$ by Eqs. (18) - (20).

10. Procedures

It would seem prudent now to review the procedures that should be followed to correlate drop size distributions across available wind tunnel data sets:

1. For the nozzle types for which a correlation already exists, the researcher should first compare the Newtonian predictions with the Newtonian equations (for example, Eqs. (18)-(20)) for the spray material in question. If the predicted drop sizes are within ten percent of the data, the assumption of Newtonian behavior (as represented in the drop size distribution) may be a good one.

2. If not, then the viscoelastic physical properties must be measured, and the non-Newtonian equations used (as developed, for example, in Eq. (33)).

3. If the nozzle has not been parameterized, then water should first be wind-tunnel tested, to develop the Newtonian functional relationship and evaluate the constants and coefficients. Then the spray material in question should be wind-tunnel tested, and a comparison of data made with the Newtonian equations just developed. If the predicted drop sizes are within ten percent of the data, the spray material may be taken as Newtonian. Otherwise, additional wind tunnel tests will be needed to develop the non-Newtonian functional relationship and evaluate the additional constants and coefficients. Physical properties will be needed to close the problem accurately.

4. It is important that accurate physical property measurements and drop size distributions be obtained. Otherwise, the correlation coefficient for data compression, reflecting all of the errors built into the data, will be low.

5. It is important that sufficient wind tunnel tests be obtained to make the least square approach discussed here work well. For all the data sets in the USDA Forest Service data base, and all other data found in the literature search, few tests (beyond water) were performed in the systematic way necessary to invoke the techniques discussed here, and enable an accurate prediction of the drop size distribution beyond the wind tunnel test conditions. Limiting wind tunnel tests must be included in any study to cover the entire anticipated range of the application for the spray material. Clearly, a minimum of eight distinct wind tunnel tests are needed to extract the four Newtonian constants (running water) and eight additional tests are needed to extract the four non-Newtonian constants. The cost of this testing must be weighed against the use of the spray material in the field application: it makes no sense to develop the non-Newtonian equation if the spray material is always applied in a specified concentration at a specified nozzle angle by an aircraft spraying at a specified speed.

11. Conclusions and Recommendations

This study has reviewed the state of Newtonian and non-Newtonian spray material characteristics, and attempted data compression with available wind tunnel data. Starting with first principles of dimensional analysis, we have developed consistent nondimensional parameters that group into well-accepted sets (Reynolds and Weber numbers, for example). We have correlated the water runs from the USDA Forest Service drop size distribution data base with a simple expression that recovers the details of the spray for conditions that were not included in the equation development. We have speculated upon the form of the non-Newtonian terms for the atomization of viscoelastic fluids.

What is now needed is a two-fold process:

1. A systematic laboratory examination of non-Newtonian fluids (and their Newtonian counterparts) should be undertaken. Benchtop testing should be considered to better understand the behavior of viscoelastic fluids, using small quantities of commercially available non-Newtonian spray materials. The development of their physical properties, particularly the proposed stress relaxation amplitude and fluid relaxation time scale constant, must be better understood.

2. A consistent set of wind tunnel tests must be performed for anticipated spray materials and field application conditions. These tests must retain the high Reynolds and Weber number regimes present in the field spray applications, and recover the smaller drop size categories (down to 10 micrometers) because these are the drops most likely to drift, and they must therefore be quantified (through $D_{0.1}$) as well as possible. The guidelines in this paper should be followed to quantify the number of tests to conduct to recover sufficient data to quantify the behavior of the tested spray material.

It is important in both recommendations that accuracy be a high priority. Any experimental deviation will carry through to the final equations developed. Also, it would be hoped that the extensive data base developed by the Spray Drift Task Force would be available for extending the analysis in this paper, and quantifying the effects suggested here.

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